Building Energy Efficiency Program

C E E R E

Center for Energy Efficiency and Renewable Energy

University of Massachusetts
Department of Mechanical and Industrial Engineering
160 Governor's Dr.
Amherst, MA 01003-9265

2-D NUMERICAL ANALYSIS OF IGU CAVITIES AT INCLINED ORIENTATION

March 2001

Part 1. Introduction

Numerical modeling of the heat transfer by natural convection across vertical and inclined glazing ~~cavity~~ cavities is of importance to …??. It can provide detailed local heat flux distribution, which is of great significance to the condensation resistance calculation of the whole fenestration system. It also provides better understanding of convection heat transfer in glazing cavities.

Until recently, little work has been done to the glazing cavity at inclined orientation. The work presented here provides benefits towards the study on the sloped skylight and similar application. It is also a topic of interest of National Fenestration Rating Council to develop a standard for rating fenestration products at inclined orientation.

In this work, three of the seven IGU cavities from the ASHRAE condensation resistance validation project-Phase I [1] was modeled at different tilt angles from horizontal (heated-from-below) to vertical (heated-from-side) orientation, using FIDAP8.52 [2] software package, in order to determine the dependence of heat transfer and flow pattern across the IGU cavity on the tilt angle. The three IGU cavities are selected, among which cavity 1 has medium aspect ratio and relatively large Rayleigh number; cavity 3 has large aspect ratio and small Rayleigh number; and cavity 6 has medium aspect ratio and medium Rayleigh number.

The description of the 3 IGU cavities modeled is summarized in the following table and figure.

Table 1. Characteristic Dimensions of the IGU cavities

IGU cavity ID	L(m)	H(m)	Aspect ratio A (L/m)
Cav1	0.0127	0.485775	38.25
Cav3	0.00635	0.485775	76.5
Cav6	0.0127	0.485775	38.25

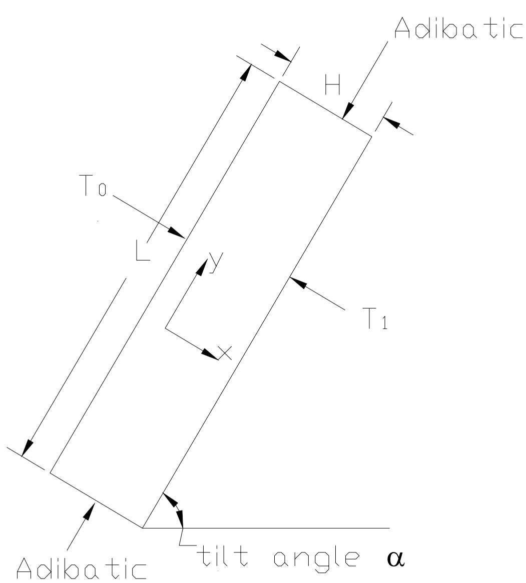

Figure 1. Geometry and boundary condition

Part 2. Problem Formulation

The geometry and boundary conditions for an IGU cavity typical of fenestration system is shown in Figure 1 where Cartesian coordinates are used. It is assumed that the cavity is infinitely long in z-direction, which is perpendicular to the plane of the drawing, to treat the problem in 2-D. The two side walls are held at constant temperatures T_1 and T_0 with $T_1 > T_0$. The top and bottom walls are adiabatic (zero heat flux).

Laminar natural convective heat transfer is governed by the set of continuity equation, momentum equation and energy equation. For the two dimensional problem considered here, following the Boussinesq approximation as well as the assumption of an incompressible fluid flow, the dimensionless form of the governing equations are given below,

Continuity equation: $$\frac{\partial \mathrm{u}}{\partial \mathrm{x}} + \frac{\partial \mathrm{v}}{\partial \mathrm{y}} = 0 \quad (1)$$

Momentum equations:

x-direction:

$$\sqrt{\frac{Ra}{\Pr}}[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}] = -\frac{\partial p}{\partial x} + [\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}] - \sqrt{\frac{Ra}{\Pr}}\Theta\cos\alpha \quad (2)$$

y-direction:

$$\sqrt{\frac{Ra}{\Pr}}[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}] = -\frac{\partial p}{\partial y} + [\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}] + \sqrt{\frac{Ra}{\Pr}}\Theta\sin\alpha \quad (3)$$

Energy equations:

$$\sqrt{Ra}\Pr[\frac{\partial\Theta}{\partial t} + u\frac{\partial\Theta}{\partial x} + v\frac{\partial\Theta}{\partial y}] = \frac{\partial^2\Theta}{\partial x^2} + \frac{\partial^2\Theta}{\partial y^2} \quad (4)$$

The terms of time derivative are not present in a steady state calculation.

The scales of velocity, temperature, pressure, length and time having been taken to make the field variables dimensionless are given by *(I can't understand what was done here!?)Please take a look at my dissertation how are non-dimensional quantities presented. Where do you define Θ?Also you need to distinguish between dimensional and non-dimensional quantities.*

$$U, \Delta T = T_1 - T_0, \frac{\mu U}{L}, L, L/U$$

respectively, where U, the reference velocity**,** is defined as $(\mathrm{g}\beta\Delta\mathrm{TL})^{1/2}$.

The boundary conditions imposed in the dimensionless form for this problem are as follows:

Temperature boundary conditions on the side walls:

$$\Theta(x=0, y)=0, \quad \Theta(x=1, y)=1 \tag{5}$$

Non-slip velocity boundary conditions on all bounding surfaces are:

$$u(x=0, y)=v(x=0, y)=0, \quad u(x=1, y)=v(x=1, y)=0 \tag{6}$$

$$u(x, y=0)=v(x, y=0)=0, \quad u(x,y=H^*)=v(x,y=H^*)=0 \tag{7}$$

Boundary conditions at the top and bottom surfaces:

$$\text{ZHF:} \quad \left.\frac{\partial\Theta}{\partial y}\right|_{y=0}=0, \quad \left.\frac{\partial\Theta}{\partial y}\right|_{y=H^*}=0 \tag{8}$$

The dimensionless parameters governing the flow behavior in a vertical IGU cavity are, aspect ratio A and Rayleigh number Ra. For a cavity at a tilt orientation, the tilt angle is another important governing parameter.

The aspect ratio A is defined by,

$$A=\frac{H}{L} \tag{9}$$

where H is the height of the cavity and L is the width of the cavity.

The Reyleigh number is given by,

$$Ra=\frac{\rho g\beta(T_1-T_0)L^3}{\mu\alpha} \tag{10}$$

The properties of ~~filled~~ cavity air are given in Table 2 for each IGU cavity. The surface temperatures T_1 and T_0 were calculated by Window4.1 program [3] and are also listed in Table 2. *(Somewhere here there should have been reference to original Yie's work)*

Table 2. Thermo-physical ~~Properties~~ properties ~~and Rayleigh number~~ of Air

IGU cavity ID	cav1	cav3	cav6
T_1 [K]	280.55	278.45	285.45
T_0 [K]	259.45	260.05	272.55
Air Properties			
Viscosity μ [kg/m-s]	1.70E-05	1.69E-05	1.74E-05
Density ρ [kg/m^3]	1.30461	1.30824	1.263
Specific heat C_p [J/kg-K]	1005.7	1005.689	1005.83

Conductivity k [W/m-K]	2.39E-02	2.38E-02	2.46E-02
Volume expansion β [1/K]	3.71E-03	3.72E-03	3.59E-03
Diffusivity α [m^2/s]	1.82E-05	1.81E-05	1.93E-05
Ra	**6559.7**	**722.9**	**3441.8**

For a cavity model with constant temperature boundary conditions on cavity hot and cold wall, the total heat transfer coefficient is, — I am not sure what is the significance of h? Can't you describe Nu in terms of q, T_1, T_0, L and k

$$h = \frac{q}{T_1 - T_0} \quad (11)$$

The average Nusselt number is calculated as: $Nu_L = \frac{hL}{k}$ (12)

where L is the cavity width and k is the conductivity of the air.

The local Nusselt number is calculated as : $Nu = \frac{hy}{k}$ (13)

where y is the distance starting from the bottom corner of the hot side wall and along it.

Using the dimensionless governing equations described above, the resulting dimensionless heat flux q^* is just the average Nusselt number. The average Nusselt number is given by,

$$Nu_L = \frac{q}{\Delta T}\frac{L}{k} = \frac{q}{k\Delta T / L} = q^* \quad (14)$$

Where $\frac{k\Delta T}{L}$ is the reference heat flux .

Part 3. Numerical Solution Methods

Both steady state and transient form of the governing equations are solved using FIDAP8.52 software package *(ref)*.

When transient governing equation was solved, the initial condition for temperature is zero everywhere in the problem domain. At dimensionless time t=0, the temperature of the right side wall suddenly changes to Θ=1 and remains for the rest of the calculation. At each iteration, the convergence of both solution vector u_i and the residual vector $R(u_i)$ is checked to guarantee the accuracy of the solutions. A technique is used to determine the total time steps needed for the transient analysis that the average Nusselt number at each time step is plotted versus time step, and when the standard deviation for the last 300 time steps is less than 0.001, the solution is assumed not changing any more.

When steady state governing equation was solved, the incremental loading method is used which is to use the result database from a simulation of a smaller

Rayleigh number as the starting point of a simulation of a larger Rayleigh number, in order to guarantee the convergence. However, around the critical tilt angle, a good convergence is hard to get.

A ~~distinguished~~ significant difference at the range of 20° to 40°on figure 2 and 3, which plot the result of the average Nusselt number from both steady state analysis and transient analysis, can be seen. This range is considered a critical transition point from one dominant heat transfer mechanism to another. Since the steady state analysis couldn't get a good convergence in this range, the steady state analysis is not a recommended method to study the heat transfer in this range. Outside this range, the steady state analysis can be a reliable method as transient analysis to find a solution easily and fast with satisfying convergence if the incremental loading method is used.

The values of mesh density used are 16x128 for cavity 1 and cavity 6 with the aspect ratio 38.25, and 16x256 for cavity 3 with the aspect ratio 76.5.

Part 4. Result and Discussion

The average Nusselt numbers at different tilt angles are plotted in Figure 2, 3, 4 respectively for cavity 1, cavity 6 and cavity 3, as well as the values calculated by ISO 15099 [5] (The data table is in Appendix.) For cavity 1 and cavity 6, both steady state analysis and transient analysis are presented.

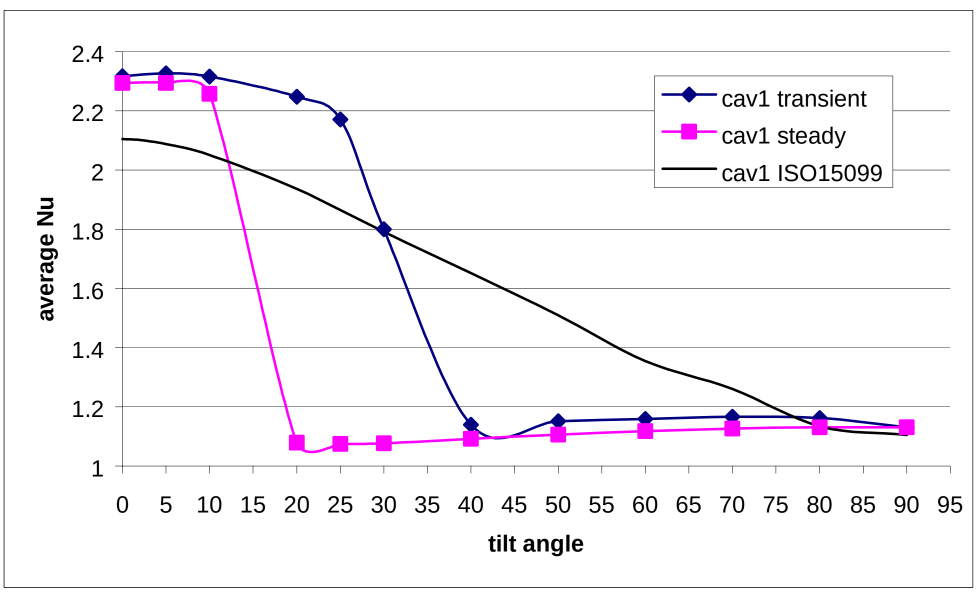

Figure 2. Average Nusselt number at different tilted angles from 0° to 90°. (cav1—A=38.25, Ra=6559.7)

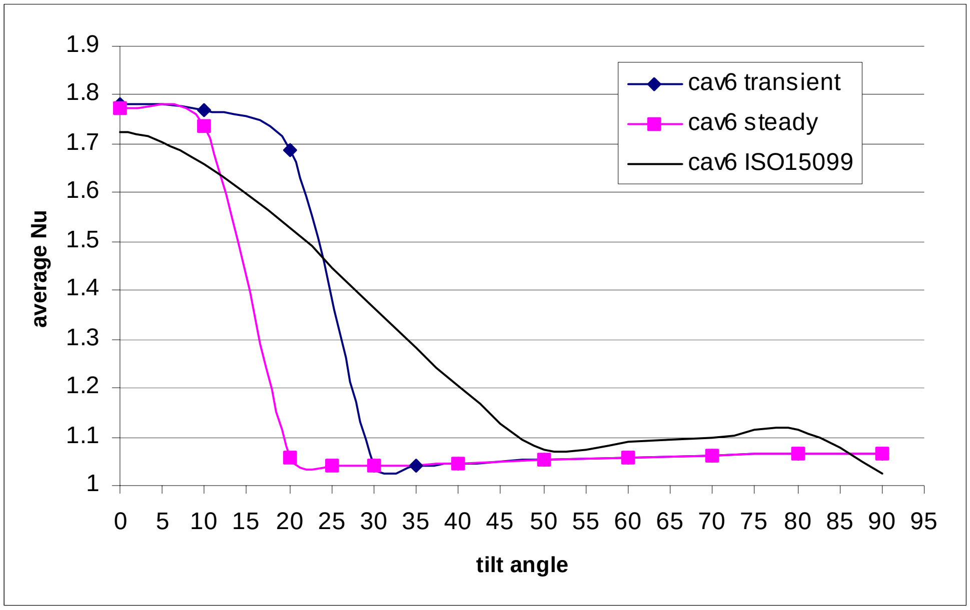

Figure 3. Average Nusselt number at different tilted angles from 0° to 90°. (cav6—A= 38.25, Ra=3441.8)

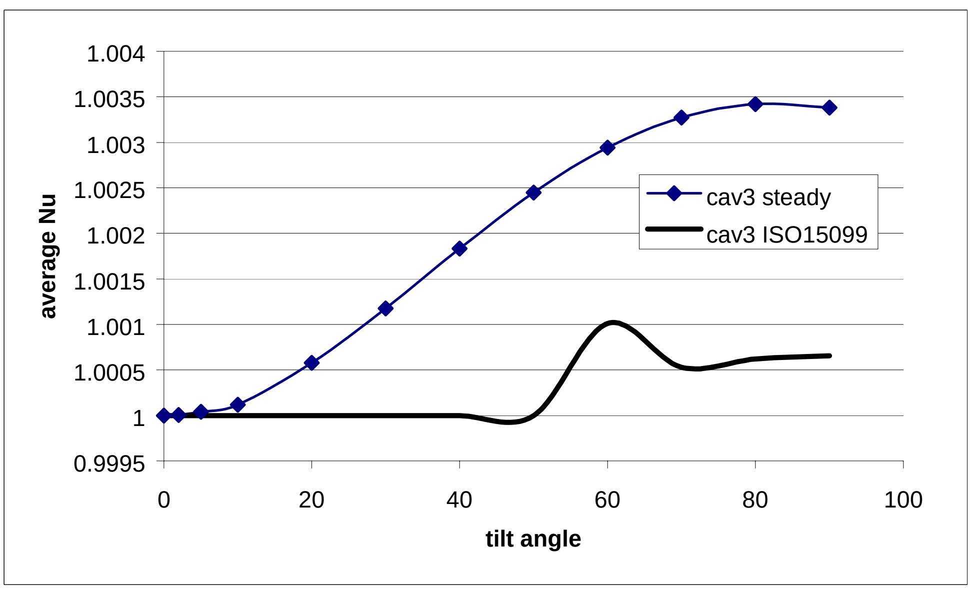

Figure 4. Average Nusselt number at different tilted angles from 0° to 90°. (cav3—A=76.5, Ra=722.9)

Since transient analysis provides more reliable results (the reason is as presented above), the following discussion is based on the result from transient analysis. You also need to indicate means of validating these numbers (i.e., other numerical studies, other experimental studies, etc.)

From the horizontal position (heated-from-below) to the vertical position (heated-from-side), the average Nusselt number decrease with the tilt angle increasing until some minimal points for cavity 1 (see figure 2) and cavity 6 (see figure 3) and then a small increase with the tilt angle increasing toward a local maximum point around 80°. The point at which the average Nusselt number reaches its minimum is different for the two cavities; it is between 40° and 45° for cavity 1, and between 30° and 35° for cavity 6.

For cavity 3 (see figure 4), the average Nusselt number keeps decreasing from 90° to 0° though the scale of this decrease is very small. This is expected as cavity 6 has a Rayleigh number which is much smaller than the generally accepted critical value Ra_c satisfying the equation $Ra_c \cos\alpha < 1708$. Below the critical Ra number Ra_c, heat is transferred across the cavity by the conduction mechanism alone [4].

The streamline contours at several selected angles are given in figure 5 and figure 6 for cavity 1 and cavity 6 respectively.

For cases that tilt angle $\alpha < \alpha_c$ (critical tilt angle at which average Nusselt number reaches its minimum), both cavity 1 and cavity 6 have the Ra number beyond the critical $Ra_{c,}$ some instability arises which is associated with what may be called a “top-heavy” situation (ref??). In the results of this work, the cavity is filled with a series of rolls rotating about the z-axis (pointing into the paper) and the rotating direction of each is contrary to the neighboring one (better wording!). These rolls persist until the tilt angle 40° for cavity 1 and only 20° for cavity 6.

Beyond the critical angle α_c, there exists fluid motion for any finite Rayleigh number. Since both cavities have different Ra numbers, the flow shows different patterns. For cavity 6 with a relatively smaller Ra number, the motion is relatively simple, consisting of one large cell which fills the whole cavity, the air rising on the hot side wall, falling down the cold side wall and turning at the opposite end of the cavity. Only around the critical angle α_c, there exist some small cells at the two ends of the cavity (see figure 6 from 30° to 40°) and these cells die out at 50°. For cavity 1, the main motion is also the single large cell filling the whole cavity. But there are cells which have thinner shapes than those appeared below the critical angle α_c filling the whole cavity either (see figure 5 from 40° to 80°). These cells persist until almost vertical orientation.

It is considered that starting from the critical angle α_c, the air experiences a transition from one dominant heat transfer mechanism to the other different dominant mechanism. But the old mechanism still has its effect on the flow pattern and the heat transfer, and this influence dies out gradually as the cavity rotates towards the vertical orientation. With a larger Rayleigh number, this dying out process lasts longer.

The local heat flux profile is related to the behaviors of these cells. Figure 7 plots the local heat flux along the hot side wall at some select tilt angle for cavity 1.

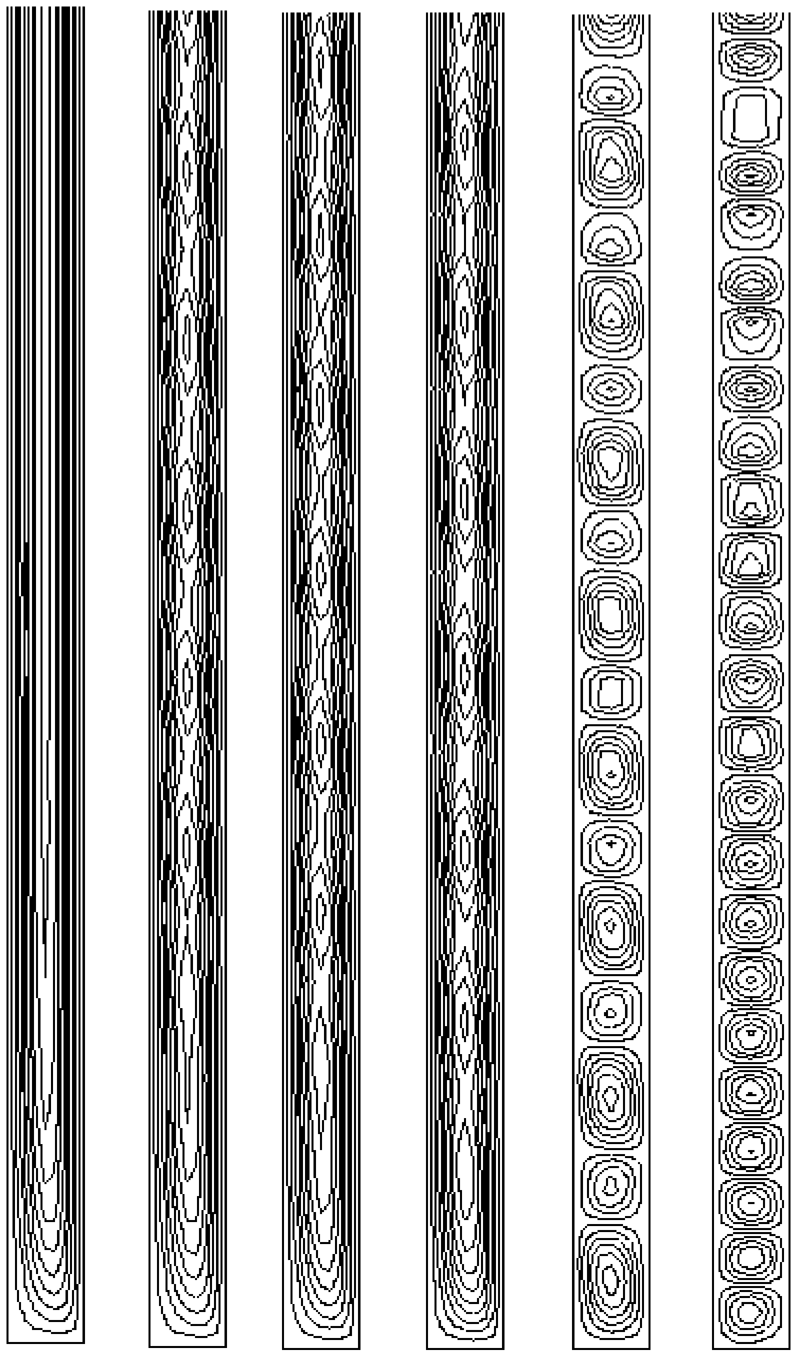

Figure 5. Cell patterns in cavity 1 at selected tilt angles (only 1/3 of the total height from the bottom)

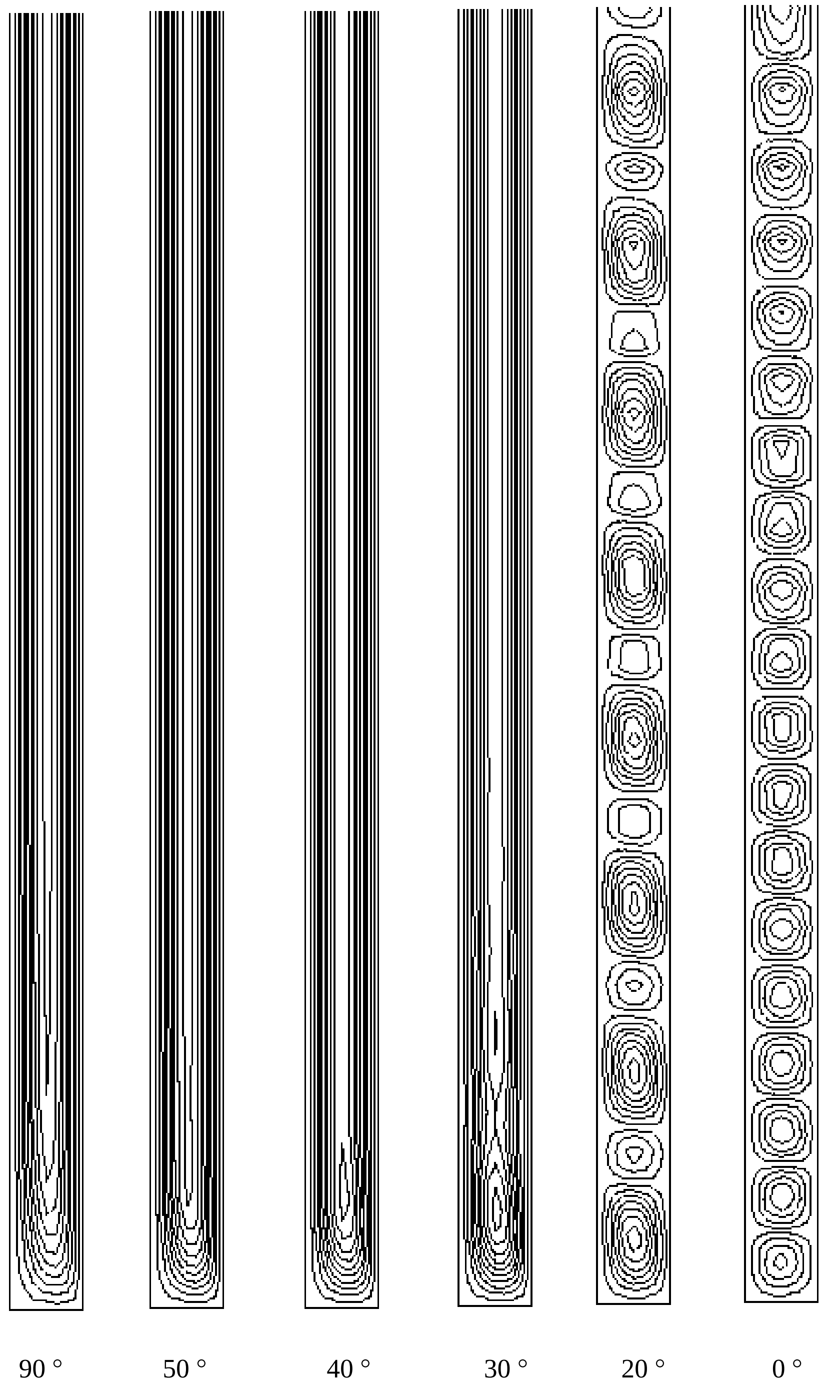

Figure 6. Cell patterns in cavity 6 at selected tilt angles (only 1/3 of the total height from the bottom)

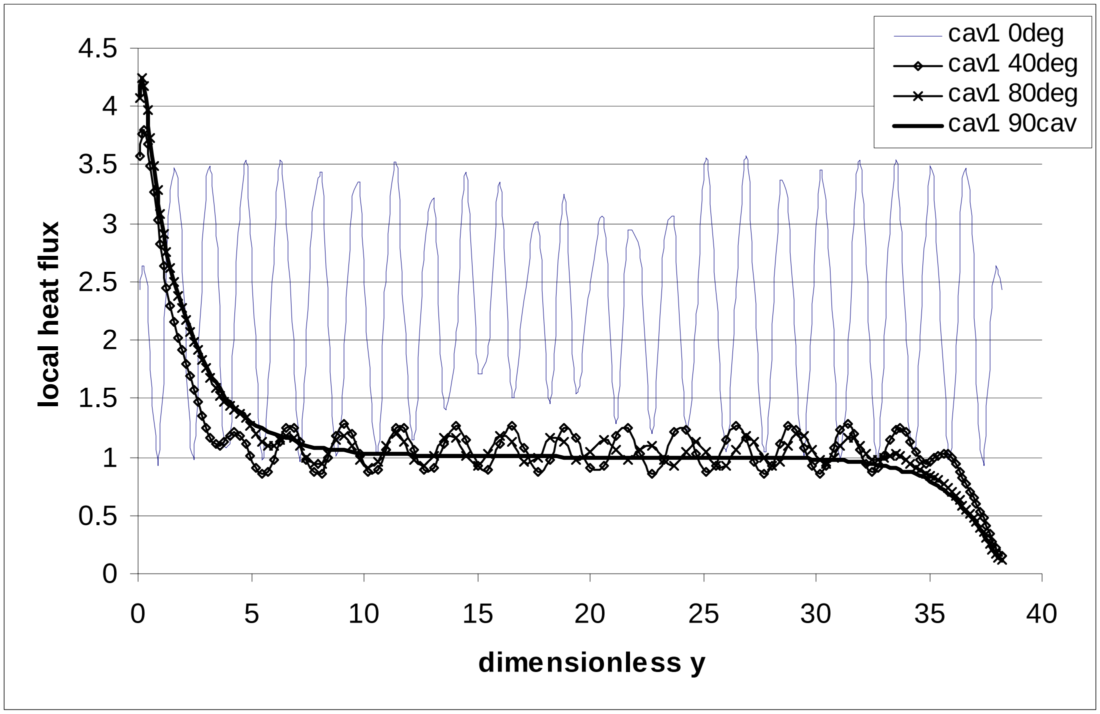

Figure 7. Local heat flux along the hot side wall for cavity 1 at selected tilt angles

Part 5. ~~Future~~ Continuation of this work

The critical tilt angle is of great significance since at this point the average Nusselt number reaches its minimal value, thus, a further study is needed to find out the factors which influence it and how they influence it. For this aim, more numerical modeling for different Rayleigh numbers and aspect ratios need to be carried out. Further discussion is necessary about validation, and extension into turbulent flow, etc.

Reference:

Format for references should be consistent to the way we did it in the past.

1. Technical Progress Report: Condensation Resistance Validation Project (Phase I)- FIDAP Modeling of Seven IGU Units

2. FDI, 2000. FIDAP 8.52 Users and Reference Manual, Fluid Dynamics International, Fluid Dynamics Analysis Package Revision 8.52

3. LBL, 1994. Window 4.1: A PC Program for Analyzing Window Thermal Performance, Lawrence Berkeley Laboratory, Windows and Daylighting Group, Berkeley, CA

4. White, F.M., 1988. Heat and Mass Transfer, Addison-Wesley Publishing Company.

5. ISO/DIS 15099, Thermal Performance of Windows, Doors and Shading Devices – Detailed Calculations

Appendix:

Appendix Table 1: Average Nusselt data

	cav1: A=38.25, Ra=6559.7			cav3: A=76.5, Ra=722.9			cav6: A=38.25,Ra=3441.8		
α	transient	steady	ISO 15099	transient	steady	ISO 15099	transient	steady	ISO 15099
0	2.31657	2.29358	2.105148	N/A	0.999998	1	1.781267	1.771197	1.725397
2	N/A	N/A	2.101399	N/A	1.000007	1	N/A	N/A	N/A
5	2.326399	2.293841	2.088107	N/A	1.000042	1	N/A	N/A	N/A
10	2.31597	2.257623	2.051291	N/A	1.000119	1	1.770872	1.736979	1.659388
20	2.2478	1.078584	1.936484	N/A	1.000577	1	1.6873	1.057799	1.526187
25	2.171215	1.074678	1.863487	N/A	N/A	N/A	N/A	N/A	N/A
30	1.80051	1.07666	1.791348	N/A	1.001175	1	1.039784	1.038874	1.363849
35	N/A	N/A	N/A	N/A	N/A	N/A	1.04249	1.039171	1.281732
40	1.1392	1.09202	1.652383	N/A	1.001832	1	1.046078	1.046094	1.203961
50	1.15122	1.106314	1.509666	N/A	1.002446	1	1.05288	1.052865	1.074837
60	1.1586	1.117928	1.354944	N/A	1.002941	1.00101	1.058396	1.058372	1.08803
70	1.16678	1.126138	1.2608	N/A	1.003272	1.00053	1.06218	1.062164	1.0999
80	1.16225	1.130496	1.1338	N/A	1.00342	1.00062	1.063972	1.063981	1.1129
90	1.13068	1.13072	1.104339	N/A	1.003383	1.000656	1.063619	1.063621	1.023694