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**Proposed Methodology For Modeling Tubular
Skylights For NFRC Rating Purposes**

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BACKGROUND:

Tubular skylights are group of products that can loosely be defined as non-standard skylight products. Their primary purpose is to provide daylighting, and not view to the outside. For this reason, there are some arguments if these [products can be considered fenestration at all. However, because they penetrate building envelope and provide some of the essential functionality of fenestration system (i.e., daylight) they are considered to be fenestration product.

Currently there is no NFRC approved methodology to test or simulate tubular skylight thermal or solar-optical performance. There were several attempts, sponsored by NFRC, to measure their thermal performance at several NFRC accredited testing laboratories. Test specimens were measured in horizontal orientation and without the shaft that is normally installed with these products. The measured thermal performance varied significantly (i.e., by 100%) between different laboratories making those measurements un-suitable for validating computer programs. In addition, it was felt that these products need to be evaluated in the vertical position and with some representative shaft length. This arguably represent more correct set of assumptions because it better matches conditions that these products are exposed to in the real house installation.

At the last task group meetings, I have been charged with the task of developing temporary simulation methodology for modeling these products. During the meeting, arguments were presented regarding standard set of assumptions and boundary conditions, under which these products are to be simulated. These assumptions are presented below:

- 14 in. shaft diameter
- 30 in. shaft length
- Standard dome mounted on 14 in. shaft
- Exterior boundary conditions applied on exterior side of the dome
- Fiberglass bat insulation applied to the exposed surfaces of the shaft
- Standard ASHRAE Attic boundary conditions applied to the exposed surfaces of the shaft insulation
- Bottom of the shaft mounted in 10 in. thick surround panel (standard surround panel material, like EPS)
- Bottom of tubular skylight covered with light diffusing plate (manufacturer supplied)

Proposed Methodology

At the most recent membership task group meeting in San Antonio, the draft of the proposed temporary methodology was presented at the research subcommittee meeting. The members present at the meeting had decided that this proposed methodology was satisfactory directed me to finalize it and to present it at the next task group meetings in Cincinnati. It was also recommended to accept this proposed methodology as a permanent solution, due to small area that these products occupy, therefore not justifying expenditures for further research and development.

Based on these recommendations, the following methodology is presented below:

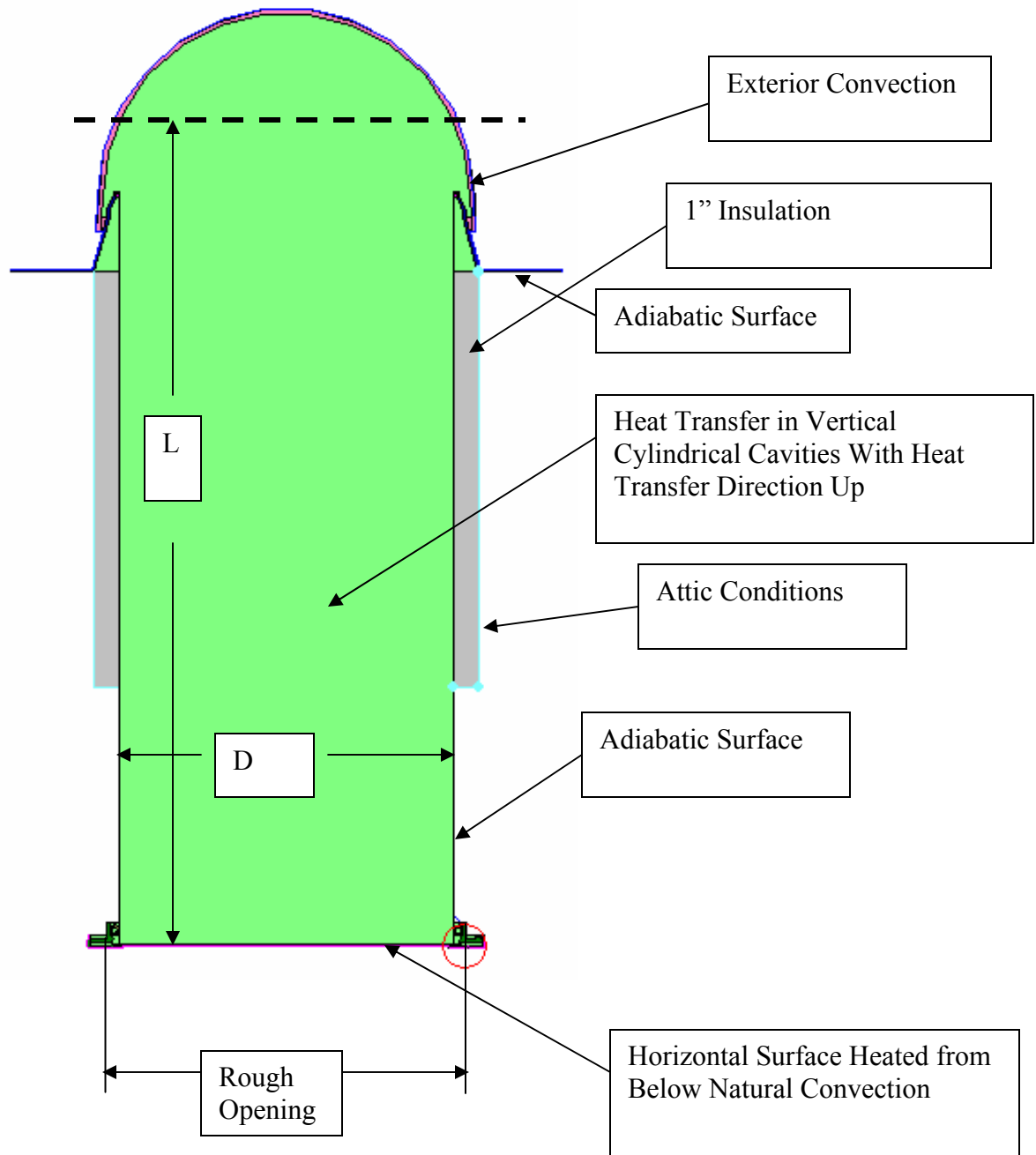


Figure 1. Typical Tubular Skylight Geometry (Courtesy of ODL) and Modeling Assumptions

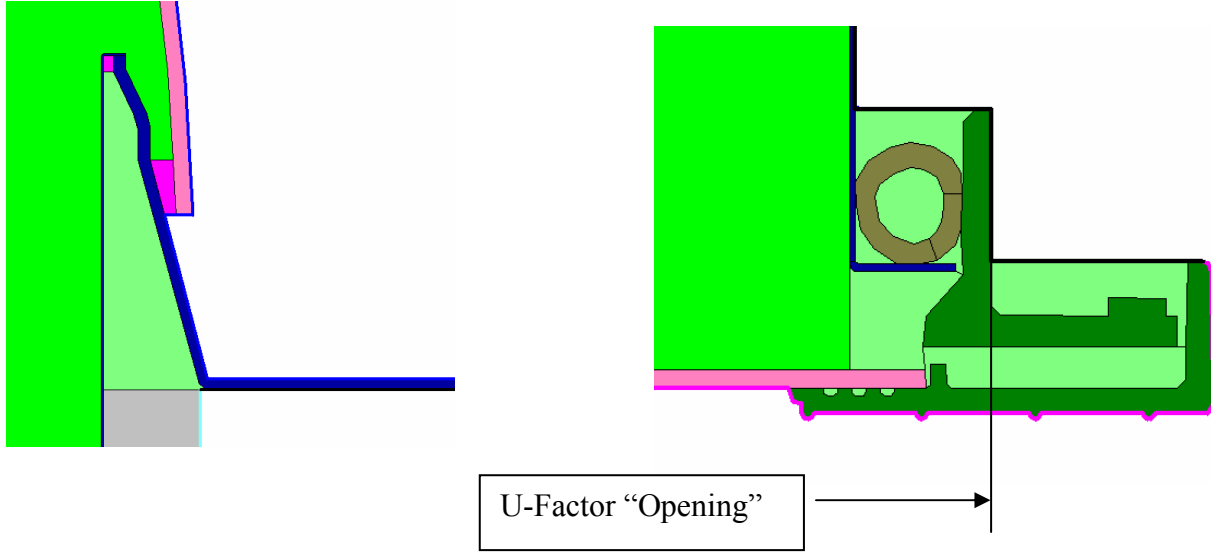


Figure 2. Details of Dome and Diffuser Plate Attachment

Heat transfer inside the shaft and dome occurs by convection and radiation. Convection heat transfer has upward direction because of temperature differential from the bottom diffuser to the top dome, where the bottom diffuser is at higher temperature than the top dome. This creates unstable conditions, where the warm air at the bottom raises due to its higher temperature and correspondingly lower density, while the colder, heavier air drops down. Because of the height of the shaft (i.e., cylinder) the flow is in fully turbulent regime.

From the available experimental studies, the following correlation for Nu by Hollands et al. () has been selected.

$$Nu = 1 + \left[1 - \frac{Ra_c}{Ra} \right] \cdot \left[k_1 + 2 \left(\frac{Ra^{1/3}}{k_2} \right)^{1 - \ln \left(\frac{Ra^{1/3}}{k_2} \right)} \right] + \left[\left(\frac{Ra}{5380} \right)^{1/3} - 1 \right] \cdot \left\{ 1 - e^{-0.95 \left[\left(\frac{Ra}{Ra_c} \right)^{1/3} - 1 \right]} \right\}$$

where:

$$k_1 = \frac{1.44}{1 + \frac{0.018}{Pr} + \frac{0.0136}{Pr^2}}$$

$$k_2 = 75 \cdot e^{1.5Pr^{-1/2}}$$

$$Ra = \frac{g\beta(T_h - T_c)L^3}{\nu\alpha}$$

$$Ra_c \cong 1086.4 \left(\frac{L}{D} \right)^4$$

Notes: * = only positive values

Ra_c formula applies to the case of adiabatic side walls, which is the closest approximation to above assumptions.

Radiative portion of the heat transfer inside the shaft is calculated from the formula given in ISO 15099 (ISO 2001):

$$h_r = \frac{4 \sigma T_{ave}^3}{\frac{1}{\epsilon_{cold}} + \frac{1}{\epsilon_{hot}} - 2 + \frac{1}{\frac{1}{2} \left[\left(1 + \left(\frac{L}{D} \right)^2 \right)^{\frac{1}{2}} - \frac{L}{D} + 1 \right]}} \quad \frac{W}{m^2 K}$$

where

$$T_{ave} = \frac{T_{cold} + T_{hot}}{2}$$

Based on the calculated Nu and h_r , the effective conductivity for the tubular skylight cavity is calculated from the following expression:

$$k_{eff} = \left(Nu \frac{k}{L} + h_r \right) \cdot L = Nu \cdot k + h_r \cdot L$$

Attic conditions were estimated from the formula and assumptions in 1997 ASHRAE Handbook of Fundamentals, Chapter 27:

$$Ta = \frac{A_c U_c T_c + (\rho C_p A_c V_c + A_R U_R + A_W U_W + A_G U_G) \cdot T_o}{A_c \cdot (U_c + \rho C_p V_c) + A_R U_R + A_W U_W + A_G U_G}$$

where:

A_c = Area of ceiling, 100 m²

U_c = U-Factor of ceiling + insulation

$$U_c = \frac{1}{\sum R} = \frac{1}{R_i + R_A + R_{ins}}$$

where:

R_i = Interior surface thermal resistance, $R_i = 1/8.85 = 0.113 \text{ m}^2\text{K/W}$

R_A = Attic surface thermal resistance, $R_A = 1/12.5 = 0.08 \text{ m}^2\text{K/W}$

R_{ins} = Insulation thermal resistance (10 in.), $R_{ins} = 30 \text{ m}^2\text{K/W}$

$$U_c = 0.033 \text{ W/m}^2\text{K}$$

$$\rho C_p = 1.2 \text{ kJ/m}^3\text{K}$$

$$V_C = \text{Air exchange rate in the Attic, } 2.5 \text{ l/s}$$

$$A_R = \text{Area of the roof, } \approx 150 \text{ m}^2$$

$$U_R = \text{U-Factor of the roof}$$

$$U_R = \frac{1}{\sum R} = \frac{1}{R_A + R_O}$$

where:

$$R_O = \text{outdoor surface thermal resistance, } R_O = 1/30 = 0.033 \text{ m}^2\text{K/W}$$

$$U_R = 8.82 \text{ W/m}^2\text{K}$$

$$A_G = \text{Glass area, } 0 \text{ m}^2$$

$$U_G = \text{Glass U-Factor}$$

Therefore,

$$T_A = -17.7 \text{ }^\circ\text{C}$$

Because this value differs from outdoor temperature by only 0.1 °C, the attic temperature is assumed to be equal to outdoor temperature -17.8 °C.

To estimate length L, the shaft length is added to the equivalent length of the dome. The equivalent length of the dome is calculated from the assumption that the volume of the shaft and dome is equal to the volume of the cylinder with the diameter D and height L:

$$L = L_{shaft} + L_D$$

Where:

$$L_{shaft} = 30 \text{ in.}$$

$$L_D = \frac{D \cdot \pi}{4}$$

$$L = 30 + 11 = 41 \text{ in.}$$

From which aspect ratio can be calculated:

$$A = 41/14 = 2.93$$

Indoor heat transfer coefficient is calculated from the equation for horizontal surface heated from below (ISO 2001):

$$h_{ic} = Nu_{in} \frac{k}{d}$$

$$Nu_{in} = 0.13 Ra_d^{1/3}$$

where:

$$Ra_d = \frac{g\beta(T_h - T_c) \cdot d^3}{\nu\alpha}$$

and where properties are evaluated at the mean fluid temperature, T_{mf} given by:

$$T_{mf} = T_i + \frac{1}{4}(T_s - T_i)$$

and where:

T_i = indoor air temperature, 21.1°C

T_s = surface temperature of the diffuser plate, 4 °C (for one particular run. This temperature is updated depending on the tubular skylight configurations)

therefore: $T_{mf} = 289.98$ K

Equivalent length d is calculated under the assumption that the area of the rough opening is equal to the area of a square whose sides are of length d . Therefore:

$$d^2 = \frac{D^2 \cdot \pi}{4}$$

where the area of the rough opening for this particular tubular skylight was 14.5 in. So equivalent length d is:

$$d = 12.85 \text{ in. [326.4 mm]}$$

Radiation portion of the surface heat transfer coefficient is estimated using usual black body radiation assumption, typical for vertical fenestration systems:

$$h_{ir} = 4 \cdot \sigma \cdot \varepsilon \cdot T_m^3$$

where:

$$T_m = \frac{T_i + T_s}{2}$$

Substituting all quantities,

$$Ra_d = 6.56 \times 10^7$$

$$Nu = 52.43$$

$$h_{ic} = 4.09 \text{ W/m}^2\text{K}$$

$$h_{ir} = 4.76 \text{ W/m}^2\text{K}$$

$$h_i = 8.85 \text{ W/m}^2\text{K}$$

The results of the THERM modeling are presented in Table 1 and graph 1. Thickness of attic insulation was varied for insulated and un-insulated shafts.

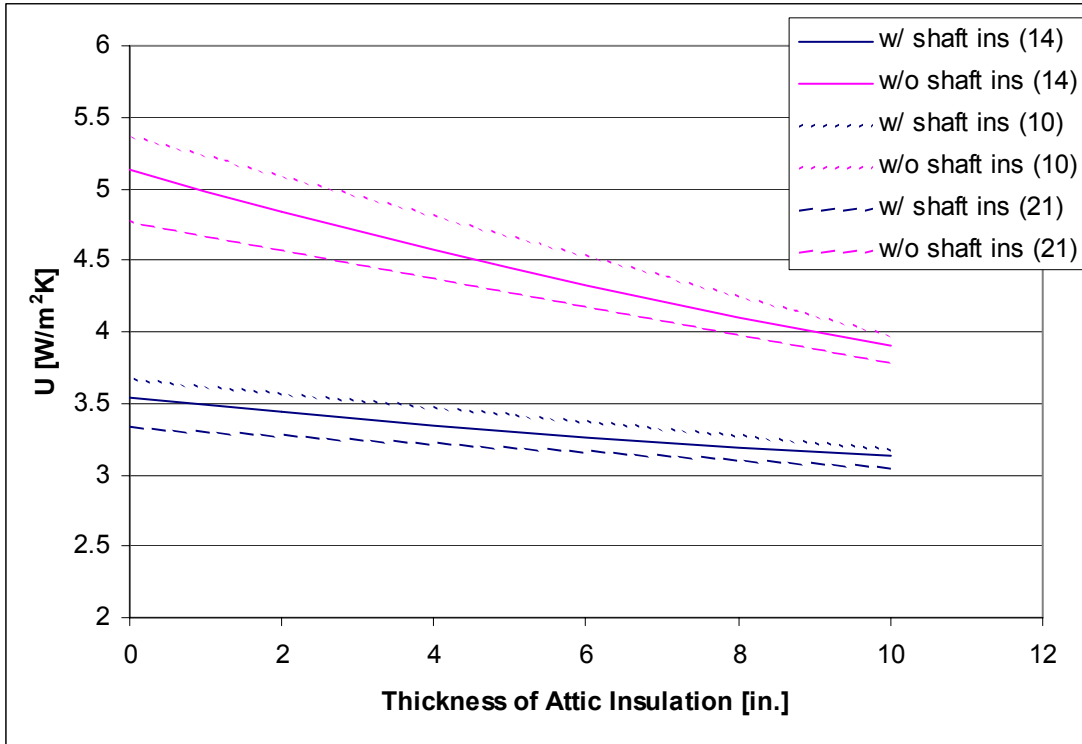


Figure 1. Change of U-Factor as a function of Attic Insulation Thickness, Shaft Insulation and Shaft Diameter (SI Units)

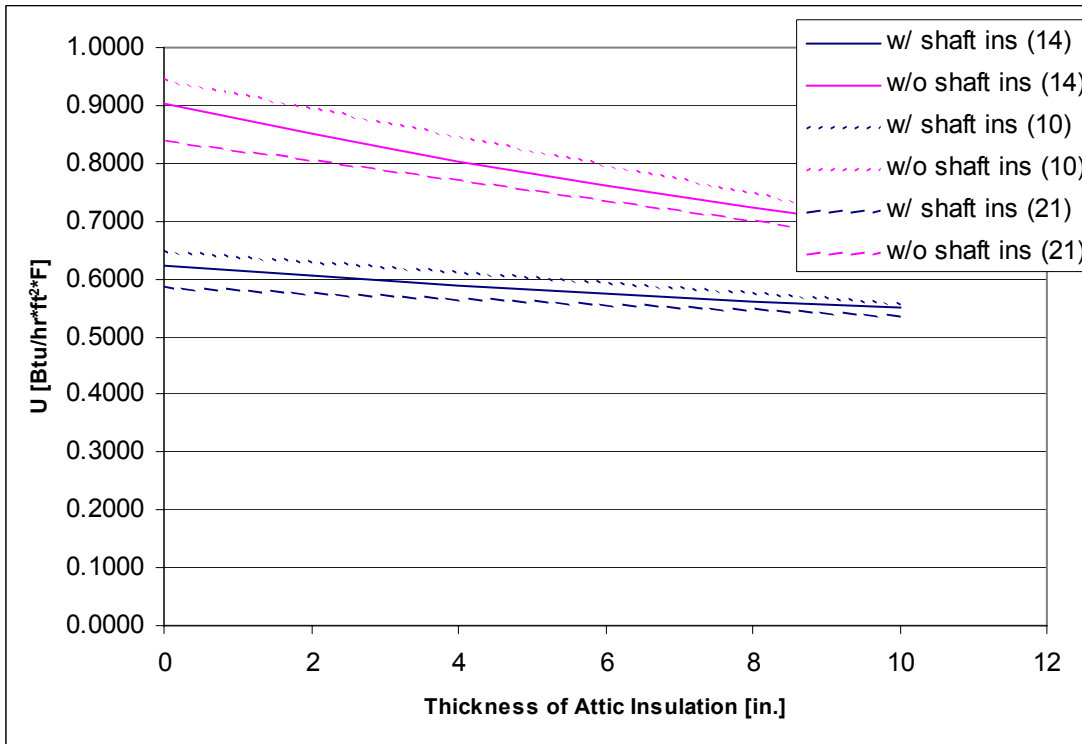


Figure 1a. Change of U-Factor as a function of Attic Insulation Thickness, Shaft Insulation and Shaft Diameter (IP Units)

Table 1. U-Factor Data of 14 in. Tubular Skylight as a Function of Attic Insulation Thickness

Attic Insulation in.	w/ shaft insulation W/m ² K	w/o shaft insulation W/m ² K
10	3.1278	3.905
8	3.1915	4.1043
6	3.2635	4.3232
4	3.3442	4.567
2	3.4344	4.8401
0	3.5387	5.1337

For all adiabatic condition on the sides of a shaft, for 14 in. shaft diameter: $U = 2.856 \text{ W/m}^2\text{K}$

Appendix A:

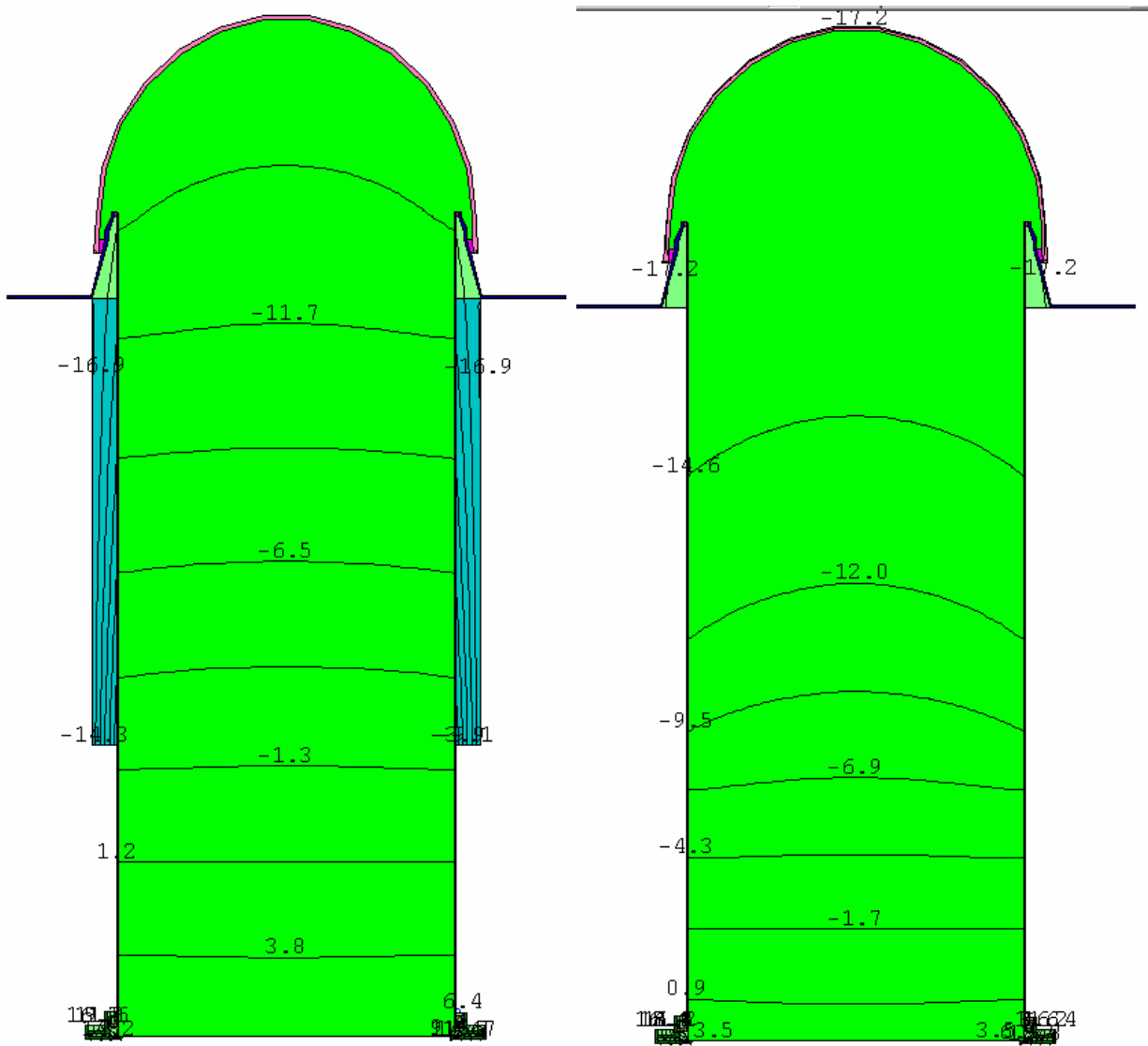


Figure A1. 10 in. Attic Insulation With and Without Shaft Insulation (14 in. Shaft Diameter)

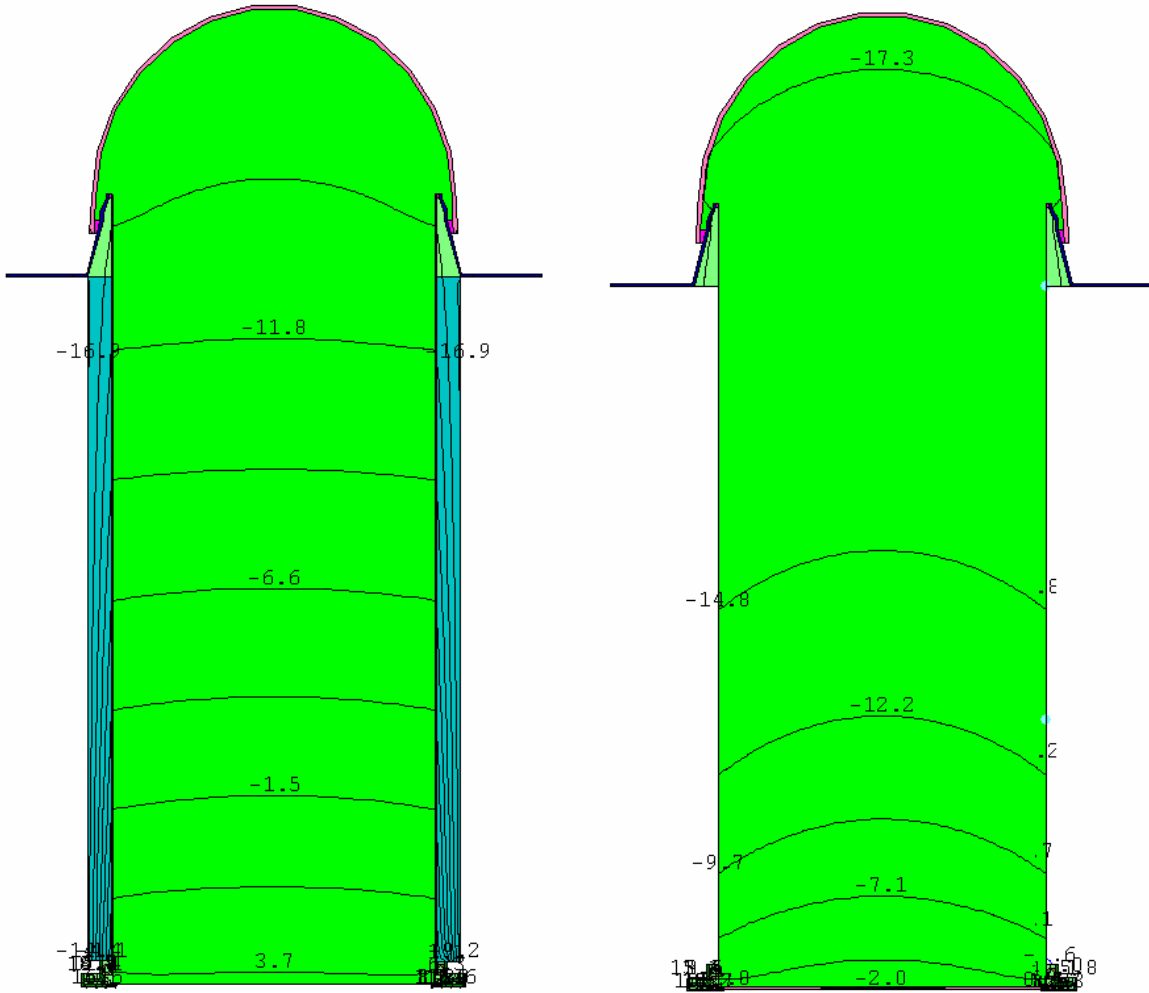


Figure A2. No Attic Insulation With and Without Shaft Insulation (14 in. Shaft diameter)

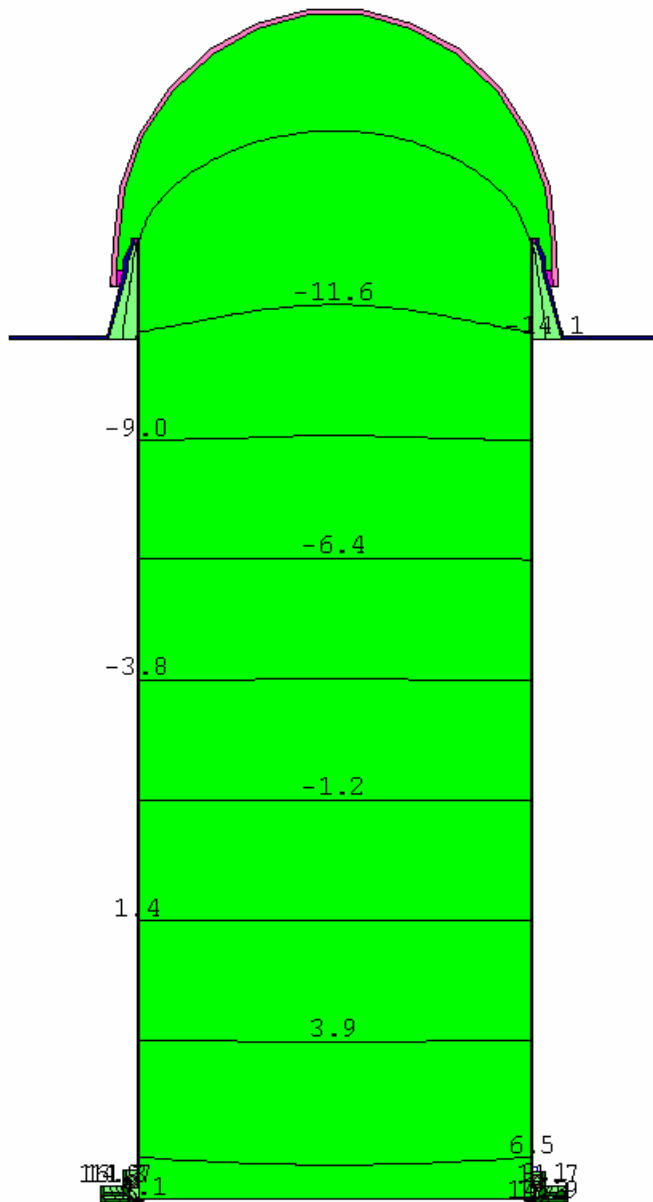


Figure A3. All Adiabatic Conditions on the Sides of a Shaft (14 in. Shaft Diameter)

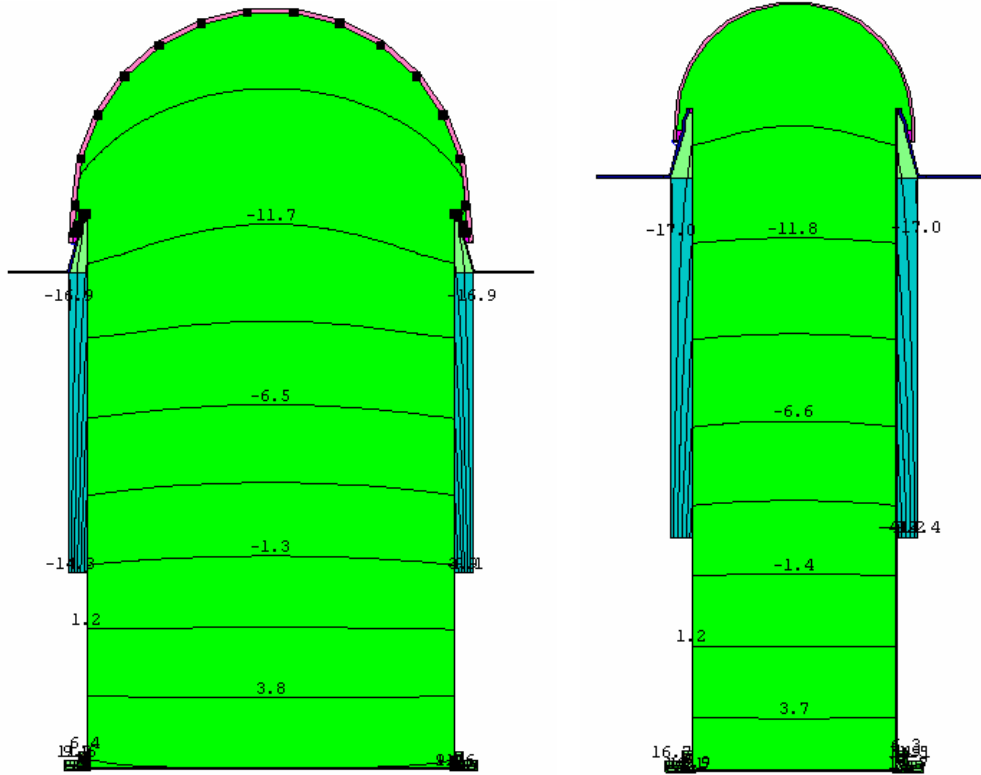


Figure A4. Heat Transfer Results for 10 in. and 21 in. Tubular Skylights with 10 in. Attic Insulation and With Shaft Insulation