Transition to Turbulence of Buoyant Flows In Vertical Confined Enclosures

1. Introduction

Buoyancy-driven natural convection in confined enclosures is receiving more and more research attention due to its wide applications. In building systems, multi-layered walls, double windows and other air gaps in unventilated spaces can be modeled as this kind of flow. Understanding the physics of flows in confined enclosures can greatly enhance the efficiency of utilization of building systems.

Depending on the conditions such as size and shape of the enclosure, the tilt angle of it and the heating applied, the flows in the enclosure can be either laminar or turbulent. While laminar flow is relatively simple and well defined, turbulent flow represents one of most complicated phenomena in the nature. According to Hinze[1] , "Turbulent fluid motion is an irregular condition of flow in which the various quantities show a random variation with time and space coordinates, so that statistically distinct values can be discerned." From this classical definition, we can tell that turbulence is time-dependent and three dimensional, as well as it is irregularly random so that statistical methods have to be used to study and analyze the characteristics of turbulence, which is much more difficult than how we deal with laminar flows. In engineering applications, turbulence often displays apparent differences than those of laminar flows. Comparatively speaking, turbulent flows often lead to much higher transport rate of momentum, energy and mass than laminar flows do. These features are widely made use of in energy systems in industry. For example, turbulence enhancers such as ribs are added to cooling systems of turbine blades and microelectronic devices to create more turbulent motions so that the overall heat transfer efficiency can be improved. However, turbulence may also be the adverse condition that people try to avoid. For instance, the enhanced heat transfer efficiency caused by turbulence in building systems usually means that more energy will be transferred between the building and its surroundings, leading to more energy waste of the building. For this reason, study of the commencement of turbulence is very important for more efficient utilization of energy. This is also the motivation of this research.

In this research, transitions from laminar flow to turbulence in confined enclosures were studied. As a very complex process, there have been quite some researchers working on this topic before. Yang[2], gave a very good summary of this process. He points out that occurrences of flow bifurcations change the structure of the flow and gradually lead to the commencement of turbulence with the change of controlling parameter such as heating. The underlying mechanism is very complicated and the description of this process needs a deep understanding of mathematics that is not available to most of the designers in engineering field. So the real purpose of this paper is to present a set of empirical equations on transitional point to turbulence with respect to the various controlling parameters of the flow. Such equations should be easily understandable by designers and applicable as a guide in their design of building systems or other systems where flows in confined enclosures are existent.

In the following sections, the basic confined enclosure flow problem will be described and its dependence on the controlling parameters will be explained. Based on the understanding of the transition process, a methodology will be formulated, based on which numerical simulations are used to calculate the transitional points for various conditions. As the last step, a set of empirical equations will then be developed to correlate the parameters to each other, which can be used as a design tool.

2. Problem description

A two-dimensional rectangular enclosure is shown in Figure 1. In real application, the enclosure is always three-dimensional. However, the size of the enclosure in the third dimension is usually very big. More importantly, there are seldom any effects such as temperature gradient or gravity applied in the third direction, which makes the flow in this direction homogeneous. So in the description of this problem, the third direction is normally neglected.

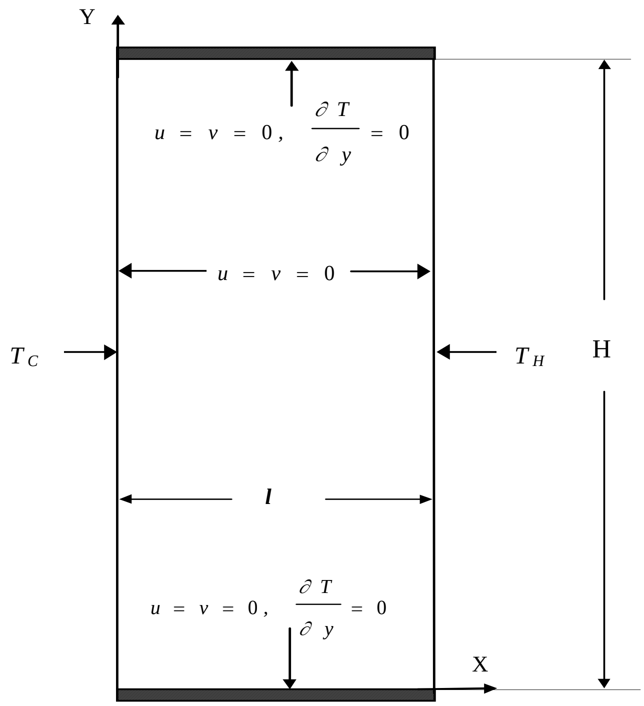


Figure 1 Confined rectangular enclosure geometry

From Figure 1, it can be seen that the two-dimensional enclosure has two horizontal sides and two vertical sides. The two horizontal sides are adiabatic and the two vertical walls have constant temperatures with $T_H > T_C$. In this condition, the density gradient of internal fluid is normal to the gravity and natural convection starts immediately when heat is applied.

For calculation purposes, it is convenient to non-dimensionalize the governing equations and to introduce the characteristic dimensionless numbers. The governing equations are non-dimensionalized using the following relations and scaling quantities:

$$u^* = \frac{u}{U}, \quad v^* = \frac{v}{U}, \quad U = \frac{\alpha}{L}\sqrt{Ra\,\mathrm{Pr}} = \sqrt{g\beta\Delta TL}$$

$$x^* = \frac{x}{L}, \quad y^* = \frac{y}{L}$$

$$\theta^* = \frac{T - T_0}{\Delta T}, \quad \Delta T = T_H - T_C$$

$$p^* = \frac{pL}{\mu U}$$

$$Ra = \frac{\rho g \beta (T_H - T_C) L^3}{\mu \alpha}, \quad \Pr = \frac{\nu}{\alpha} = \frac{\mu C_p}{k}$$

$$t^* = \frac{tU}{L}$$

The Boussinesq approximation as well as the assumption of an incompressible fluid flow are applied. The Governing equations in non-dimensional form are:

Continuity equation: $$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0 \tag{1}$$

Momentum equations:

x-direction:

$$\sqrt{\frac{Ra}{\Pr}}[\frac{\partial u^*}{\partial t^*} + u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*}] = -\frac{\partial p^*}{\partial x^*} + [\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}}] - \sqrt{\frac{Ra}{\Pr}}\theta^* \tag{2}$$

y-direction:

$$\sqrt{\frac{Ra}{\Pr}}[\frac{\partial v^*}{\partial t^*} + u^* \frac{\partial v^*}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^*}] = -\frac{\partial p^*}{\partial y^*} + [\frac{\partial^2 v^*}{\partial x^{*2}} + \frac{\partial^2 v^*}{\partial y^{*2}}] + \sqrt{\frac{Ra}{\Pr}}\theta^* \tag{3}$$

Energy equation:

$$\sqrt{Ra \Pr}[\frac{\partial \theta^*}{\partial t^*} + u^* \frac{\partial \theta^*}{\partial x^*} + v^* \frac{\partial \theta^*}{\partial y^*}] = \frac{\partial^2 \theta^*}{\partial x^{*2}} + \frac{\partial^2 \theta^*}{\partial y^{*2}} \tag{4}$$

The boundary conditions imposed in the dimensionless form for this problem are as follows:

Temperature boundary conditions on the side walls:

$$\theta^*(x^* = 0, y^*) = 0, \quad \theta^*(x^* = 1, y^*) = 1 \tag{5}$$

Non-slip velocity boundary conditions on all bounding surfaces are:

$$u^*(x^* = 0, y^*) = v^*(x^* = 0, y^*) = 0 \tag{6}$$

$$u^*(x^* = 1, y^*) = v^*(x^* = 1, y^*) = 0 \tag{7}$$

$$u^*(x^*, y^* = 0) = v^*(x^*, y^* = 0) = 0 \tag{8}$$

$$u^*(x^*, y^* = H^*) = v^*(x^*, y^* = H^*) = 0 \tag{9}$$

where H^* is the dimensionless height of the cavity.

Boundary conditions at the top and bottom surfaces:

$$\text{ZHF:} \quad \left.\frac{\partial \theta^*}{\partial y^*}\right|_{y^*=0} = 0 \,, \quad \left.\frac{\partial \theta^*}{\partial y^*}\right|_{y^*=H^*} = 0 \tag{10}$$

The dimensionless parameters governing the flow behavior in a vertical IGU cavity are, aspect ratio A and Rayleigh number Ra.

The aspect ratio A is defined by,

$$A = \frac{H}{L} \tag{11}$$

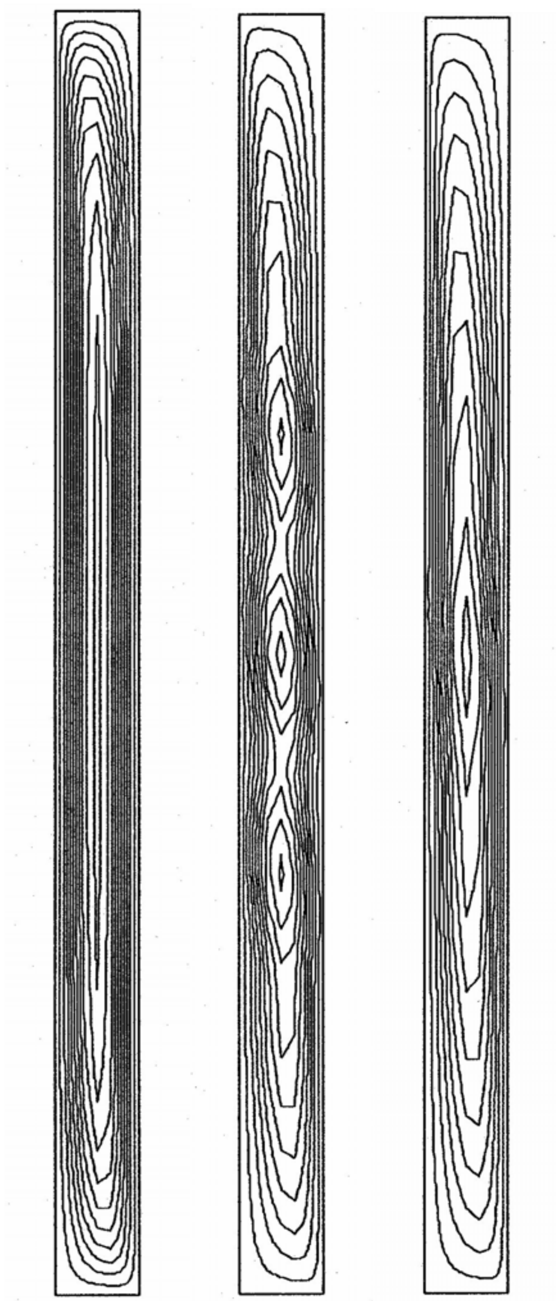
where H is the height of the cavity and L is the width of the cavity.

Pr is the property of the fluid and it was chosen as 0.71 for air, while Ra incorporates the influences of fluid property, external thermal boundary condition and domain geometry into a single parameter. Based on the governing equations, it can be seen that Ra is the main determining parameter in this problem since it represents the driving force in the enclosure – buoyancy, without which there will be no turbulence in the domain. The greater Ra is, the greater buoyancy effect will be and the flow tends to be more turbulent.

When Ra is small, which means there is only very weak buoyancy acting on the flow, it can be expected that the flow is laminar and heat transfer in the cavity is dominated by the heat conduction. However, the heat transfer by convection cannot be neglected. The convective motion of the flow creates a cell that circulating inside the enclosure. The streamlines of it are presented in Figure 2a with Ra = 5000. The aspect ratio of this cavity is 15.

As Ra increases, an important phenomenon for buoyancy driven cavity flow, multi-cellular condition, will occur, as shown in Figure 2b. The Ra under this condition is 17750. There have been intensive investigations on this topic. Batchelor[3], used linear hydrodynamic stability theory to explain the origin and characteristics of this phenomenon. Power[4], applied numerical methods to predict the transitional Ra with respect to aspect ratio A in the cavity described as above. An interesting feature under multi-cellular condition is that the local heat flux profile in the central part on either side of the cavity is not constant as when the flow is in single-cellular regime. The heat flux profiles of both conditions are shown in Figure 3.

Although the flow in the cavity can become multi-cellular, this condition cannot stay forever for aspect ratio below 30. When the buoyancy force is further strengthened all the small cells will collapse into a single strong cell again. This is the ending point of multi-cellular regime and this topic was also studied by Zhao[5]. The streamlines of this condition are shown in Figure 2c with Ra = 28750.



(a) (b) (c)

Figure 2. Cells in a buoyancy driven cavity

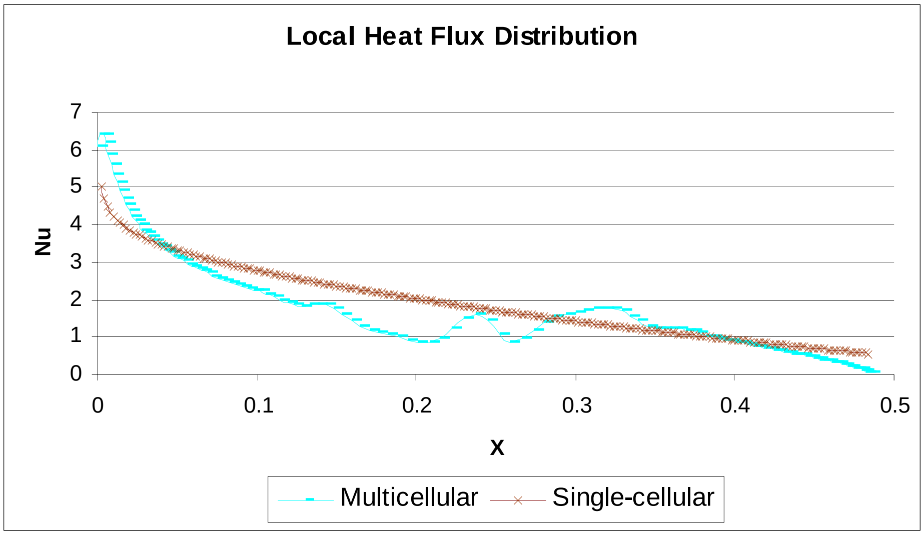


Figure 3. Local heat flux difference between single-cellular and multi-cellular

The above paragraphs discuss about the characteristics of the process of the buoyancy driven natural convection flow experiences as the driving force gradually increases. It seems that this is not very closely related to the topic of this research – transition from laminar flow to turbulence. However, the flow characteristics are closely related with the way to be utilized to discern the turbulent flow pattern in the cavity. And this will become obvious in the discussion in next section.

In this research, the transitional point from laminar flow to turbulence in terms of Ra is developed because Ra is the driving force of the flow. However, as can be seen above, aspect ratio A is also an important factor that decides the characteristics. Since all the other parameters are fixed, the derived equation will be the Ra Vs. A form.

3. Principles

The most important feature of turbulence is its randomness. So it is assumed that this method can be applied to find the transitional point from laminar flow to turbulence in such a buoyancy-driven cavity. The basic underlying idea is that when the flow goes turbulent, some temporary fluctuations can be created and detected numerically by calculating the change of overall heat transfer effect with respect to time so that the flow can be judged to have crossed the transitional point.

After the time-dependent governing equations are solved, data are collected and overall heat transfer effect is computed to observe the fluctuation. The Nusselt number Nu is defined to quantify the heat transfer effect. Nu has the form as $Nu = \frac{hL}{k} = \frac{qL}{\Delta Tk}$, which is the ratio between convection heat transfer to the conduction heat transfer, while q is the heat transfer rate. At lower Ra, the Nu is almost a flat line with respect to time, which means there is rarely any fluctuation in the flow. However, as Ra is increased, more fluctuations in Nu can be observed corresponding to the fact that the flow tends to be more turbulent with higher Ra. A good example is given in Figure 4. The ratio between the standard deviation of Nu and the mean value of Nu over a long range of time is used to detect the fluctuation.

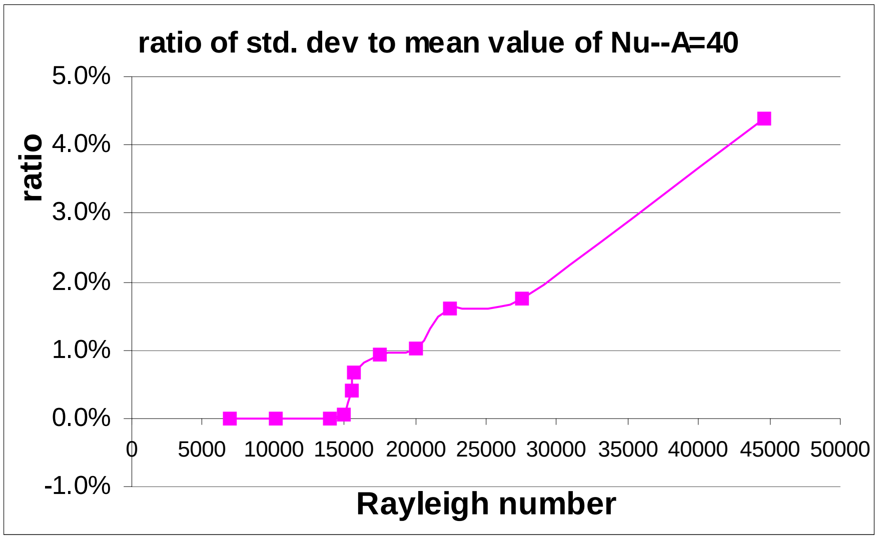


Figure 4. Fluctuation Vs. Ra

It can be seen from Figure 4 that there is no significant fluctuation at lower range of Rayleigh number. However, as Ra increases, a sudden increase of fluctuation level can be detected at Ra = 16000. Further investigation shows that before transition to turbulence, the ratio between standard deviation of Nu to the mean value of Nu is on the order of 0.01%. After it reaches the transitional point, the ratio is at the level of 0.1%. So the flow can be judged to be turbulent if this ration reaches 0.5%.

The method above works generally very well for cavities with a wide range of aspect ratios. However, if A is smaller than 30, the method is very hard to be used to detect the transitional point. Some small fluctuations are observed at first. But without increasing, the fluctuations diminish to almost zero after a certain Ra value. The explanation for this phenomenon is related with the multi-cellular phenomenon in the cavity.

It is observed that if the flow is in multi-cellular regime, the local heat flux along the vertical sides fluctuates quite significantly, indicating that the cells have different thermal characteristics with respect to each other. After the flow goes turbulent, the random motion of the cells, which are caused by turbulence, will make the overall heat transfer unstable. And this is the source of the fluctuations that are observed. If the flow is laminar, fluctuations also exist for multicellular regime, however, to a very insignificant level. This explains why some small fluctuations can be observed before they are amplified by orders of 10.

So the real reason why fluctuations can be observed for cavities with aspect ratio A larger than 30 is that the flows in these cavities are still multi-cellular when it transits to turbulence. The multicells and the turbulence combine to show the fluctuations in Nu. If aspect ratio A is less than 30, the flow in the cavity will experience the transition from single-cell to multi-cells. However, it will exit multi-cellular condition and returns back to single-cell before turbulence can be reached. Single cell in turbulence regime does not produce any fluctuations in Nu and this method will not work for cavities with A under 30. Another method has to be found to detect the onset of turbulence. In this research, two sets of simulations are run to solve for Nu for each cavity with A less than 30 at every Ra it experiences. One simulation is using laminar model while the other one incorporates a $\kappa - \omega$ turbulence model. Two results are compared and if the difference between each other is larger than 3%, the flow is regarded to be turbulent. The difference is defined as $(Nu_{tur} - Nu_{lam})/Nu_{lam}$. So in this research, the method described formerly is used to detect the transitional point when A is larger than 30 and the latter method is applied for cavities with A under 30.

4. Results and analysis

(1) A = 20

For cavity with A = 20, the ratio between standard deviation in Nu and Nu mean value is calculated and presented. However, it is found that it is impossible to use this method to detect the transitional point, as explained in the former section. So two simulations, one laminar and the other turbulent, are run to detect the onset of turbulence.

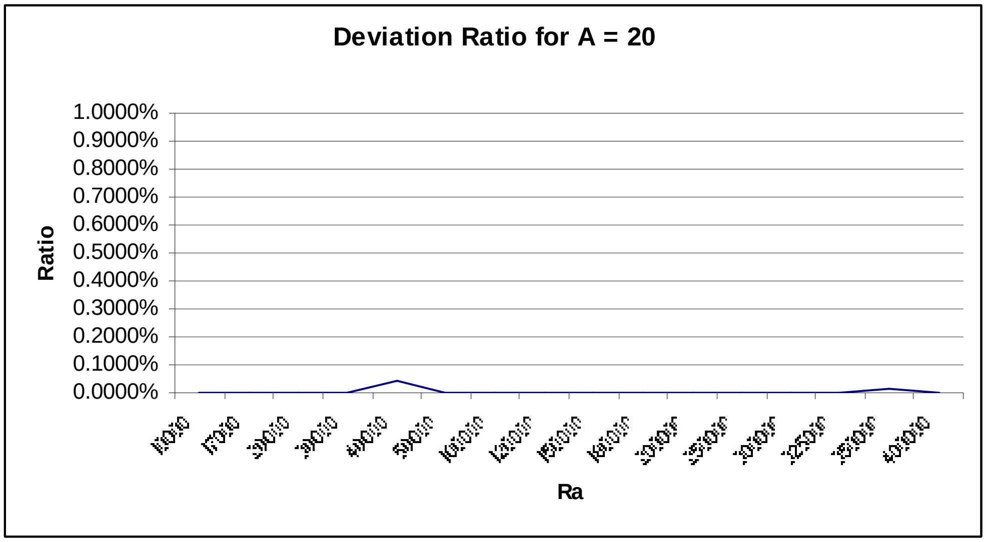


Figure 5. Standard deviation/mean value at A = 20

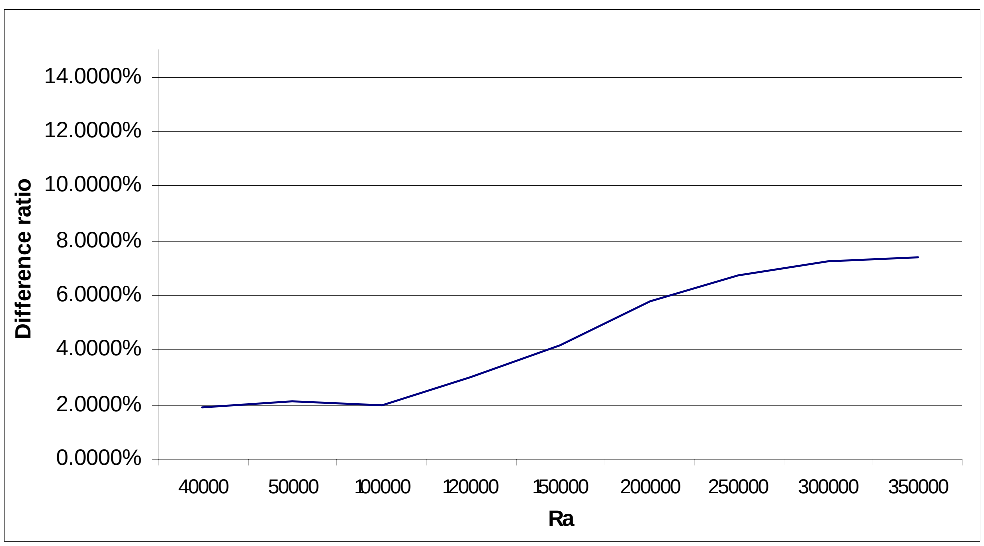


Figure 6. Difference between Laminar & Turbulent models at A = 20

(2) A = 22.5

It is also found that the method of detecting the fluctuation in Nu fails for this aspect ratio. So does it for A = 25, 27.5 and 30. So only latter method of comparing results from different models will be utilized for these aspect ratios.

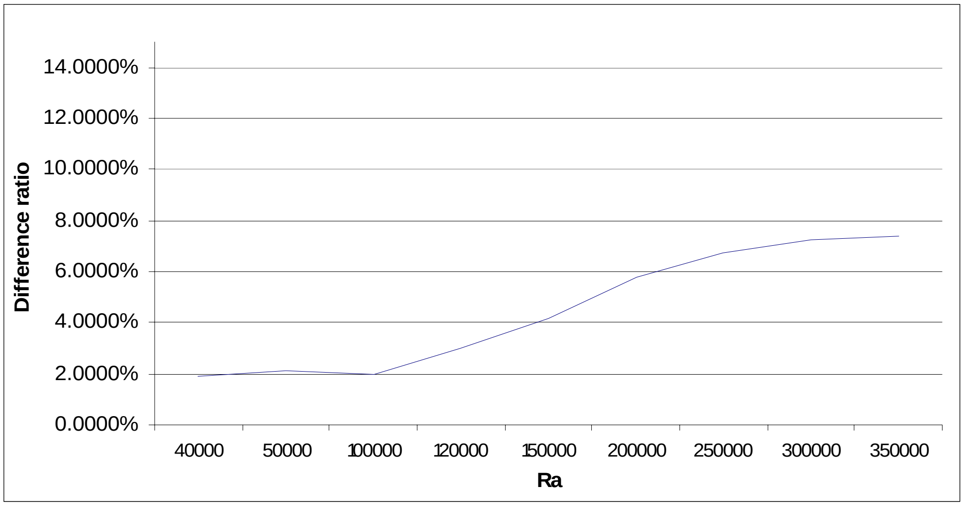


Figure 7. Difference between Laminar /Turbulent models at A = 22.5

(3) A = 25

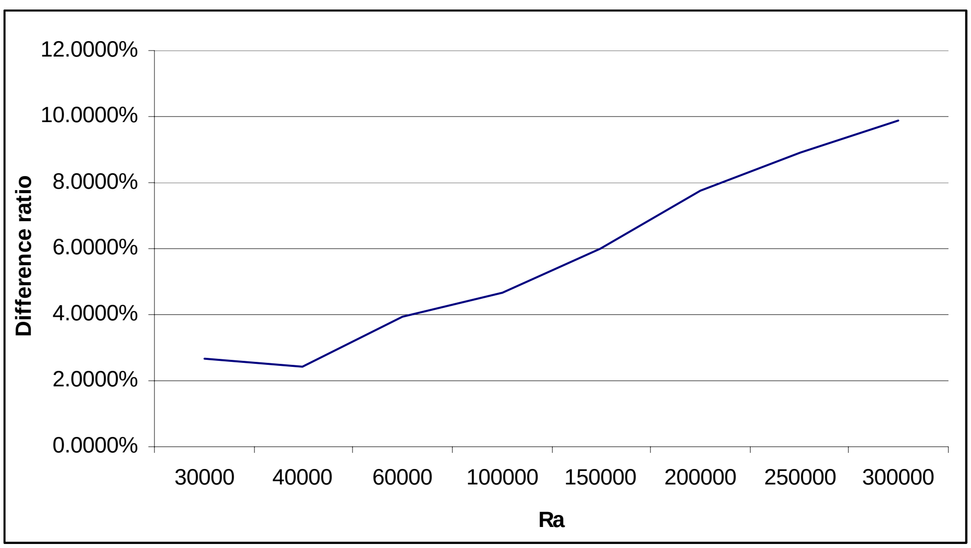


Figure 8. Difference between Laminar &
Turbulent models at A = 25

(4) A = 27.5

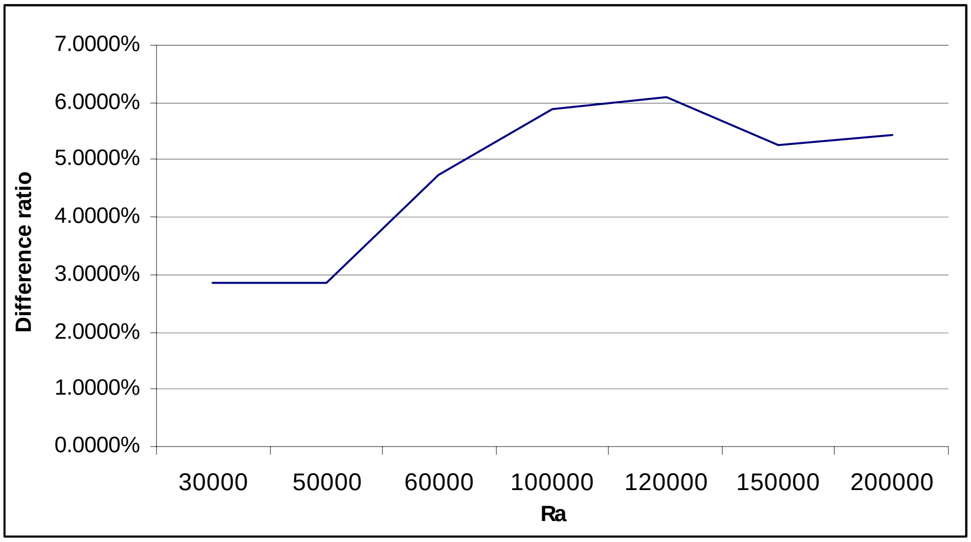


Figure 9. Difference between Laminar & Turbulent models at A = 27.5

(5) A =30

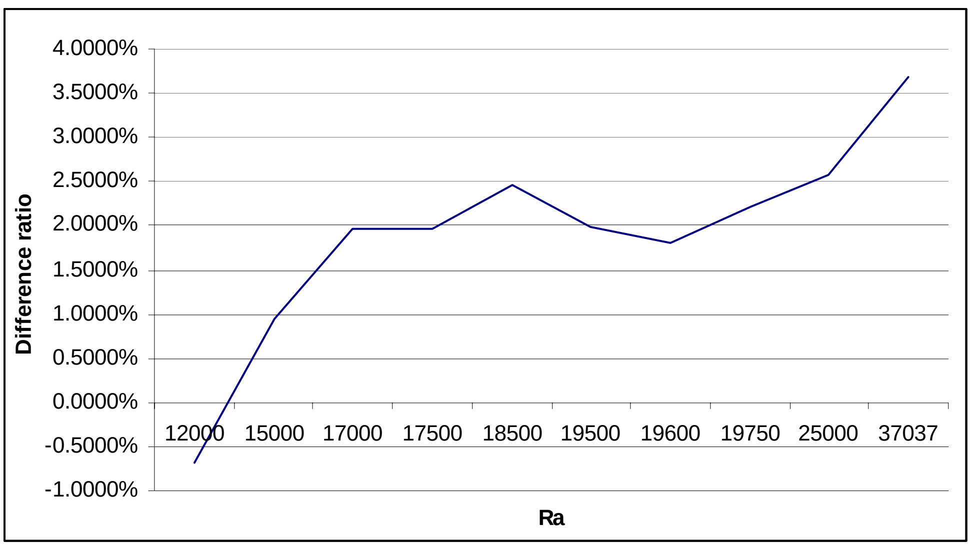


Figure 10. Difference between Laminar & Turbulent models at A = 30

(5) A = 33

For cavity with aspect ratios higher than 30, fluctuation in Nu will be used as the indicator of the onset of turbulence.

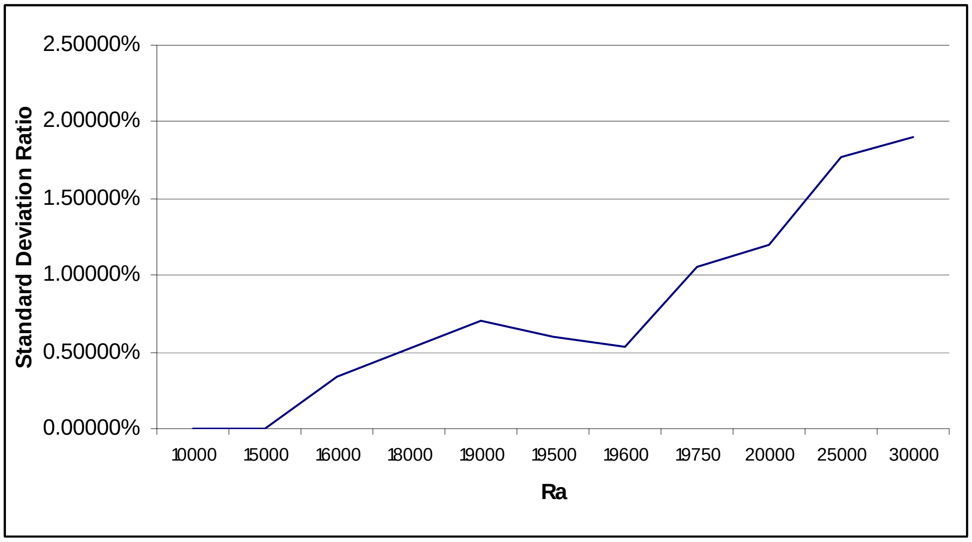


Figure 11. Standard deviation/mean value at A = 33

(6) A = 40

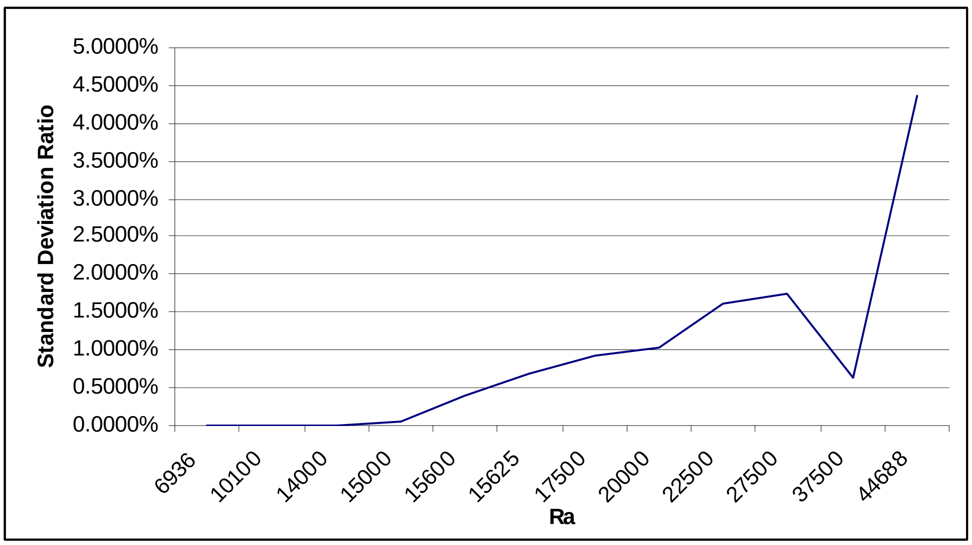


Figure 12. Standard deviation/mean value at A = 40

(7) A > 40

For aspect ratios larger than 40, detailed results will be omitted since they are similar to what obtained for A = 33 and 40.

The range of study on aspect ratios of this research is from 20 to 80 so far. So after studying the standard deviation and mean value ratio for aspect ratios above 30 and the relative difference between the results obtained from laminar model and turbulent model for aspect ratios below 30, critical Rayleigh number Ra_c for each aspect ratio is determined. The plot of $\log(Ra_c)$ Vs. $\log(A)$ is shown as below:

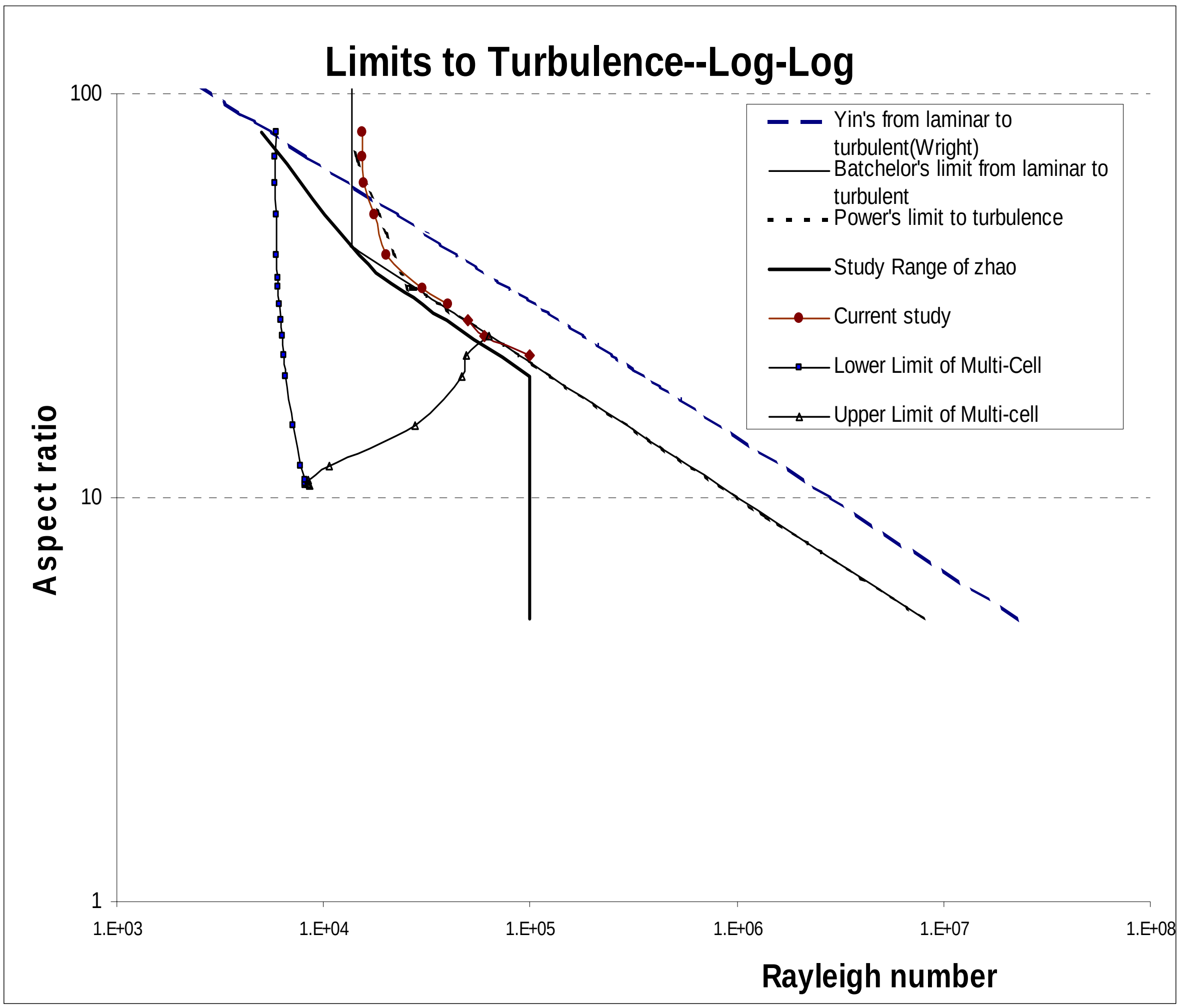


Figure 13. Critical Rayleigh Number Vs. Aspect Ratio

Based on Figure 13, the transitional Rayleigh number obtained in this research matches that of other researches well. Lower and upper limits of multi-cellular regime are included in the figure. It can be seen that for relatively lower A smaller than 30, the upper limit is smaller than the transitional Ra. Meanwhile, at higher A, the transitional Ra lies between lower and upper limit. This agrees quite well with what has been discussed about the relationship between multi-cellular regime and transitional point. The data obtained look very smooth over the whole range of aspect ratios studied and some further study will be able to generalize the data into an empirical equation at all aspect ratios, instead of using a set of equations.

5. Conclusions

Simulations of buoyancy driven natural convection flow in cavities are run to detect the transitional point in terms of Ra from laminar flow to turbulent flow under a wide range of aspect ratios. The purpose of this research is to provide a design guide for designers on fenestration systems or other building systems. Due to the complexity of the flow itself, the characteristics of the flow are discussed, based on which two methods were put forward to study on the transitional point for cavities in two range of aspect ratios respectively. The data obtained up to now is reasonable and promising in that all the data can be fit into a single empirical equation. Further research will be working on that part of the project.

Reference:

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