Center for Energy Efficiency and Renewable Energy

C E E R E

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Progress Report

Turbulent Natural Convective Heat Transfer in Vertical Rectangular Cavities

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1. Introduction

Natural convection in vertical enclosures has wide applications in building systems, such as multi-layered walls, double windows and other air gaps in unventilated spaces.

There have been several experimental studies performed on turbulent natural convection heat transfer in a cavity. However most of these studies were for cavities with aspect ratios not as high as those of typical fenestration systems, which are of our major concern in this research. Some studies covered aspect ratios as high as 80, but with very limited Rayleigh number range for these high aspect ratios. Table 1 summarizes the ranges of both aspect ratio and Rayleigh number in previous studies.

Table 1: Applicable Range of Heat Transfer Correlations

Investigator	Range of A	Range of Ra
Jakob (1967)	3.12 to 78.7	1.42×10^4 to 7.81×10^6
Eckert and Carlson (1961)	2.5 to 46.7	Defined in their figure
Elsherbiny et.al. (1982)	5,10,20,40,80,110	Each A corresponding to a range
de Vahl Davis (1968)	2.5 to 35.0	7.10×10^3 to 2.13×10^5
Yin, et al (1978)	4.9 to 78.7	1.07×10^3 to 4.97×10^6
Raithby and Wong (1981)	2.0 to 80.0	1.0×10^3 to 1.0×10^5
Zhao (1997)	5.0 to 80.0	Defined in their figure
Wright (1996)	Based on Elsherbiny(1982)	Based on Elsherbiny (1982)
Power (1999)	5.0 to 80.0	Each A corresponding to a range

In this study, the aspect ratio studied ranges from 30 to 80, which is typical of fenestration systems. Rayleigh number covers up to 200,000. Turbulent natural convective flow is modeled by the finite element program FIDAP. Based on these numerical heat transfer results, a correlation of average Nusselt number as a simple function of Rayleigh number and aspect ratio is developed and presented. Compared with the experimental method, numerical method is much more convenient and efficient. It is believed that numerical modeling and simulation will become more and more popular in thermal and fluid analysis. Meanwhile, with extended range in Rayleigh number and aspect ratio studied, designers of fenestration systems will benefit from the new correlation and this is the motivation of this research.

2. Problem Description

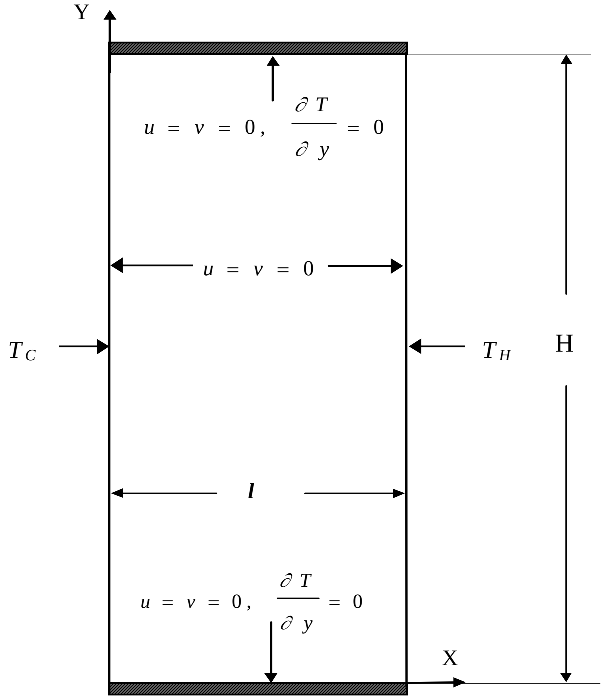

Figure 1. The geometry and boundary conditions of a vertical rectangular cavity

Figure 1 shows a two-dimensional enclosure with two horizontal adiabatic sides and two vertical sides. The two vertical walls have constant temperatures with $T_H > T_C$. Air or some other gas is usually enclosed in the gap between two vertical walls. Rayleigh number Ra is often used to characterize the flow in the enclosure. Ra is usually depicted as the driving force in the flow and it shows the relative magnitude of buoyancy force with respect to inertial force. With low Rayleigh number, flow is usually laminar with lower heat transfer rate, which is characterized by average Nusselt number (Nu). After experiencing a transitional regime, the flow will become turbulent with the increase of Ra. Meanwhile, the turbulent flow brings about a thinner thermal boundary layer compared with laminar flow, which can account for the higher Nu in turbulent regime. So it is of great importance to study the characteristics of turbulent heat transfer in the enclosure.

Although it is theoretically possible to directly apply the conservation equations to the entire turbulent flow field, it is unreasonably difficult to do so in practice. Turbulent flows involve entangled eddies the sizes of which covers a wide range of length scales. To resolve an entire turbulent flow field by direct application of the conservation equations, it is necessary to employ a computational mesh with the sizes smaller than the smallest eddies. Such meshes are extremely dense, which result in models that are very expensive to solve. To create a usable numerical model of a turbulent flow field, it is necessary to describe turbulent motion in terms of averaged quantities. Models that are based on averaged quantities characterize turbulent flows using meshes of reasonable density; therefore, they result in reasonable computational times and costs.

In order to obtain a numerical solution for turbulent flow, Reynolds decomposition is applied to the Navier-Stokes equations, which decomposes the turbulent variables into instantaneous (fluctuating) and mean (time averaged) components, and averages the flow equations over a certain time scale. This time scale should be long compared with the instantaneous motion of the turbulent flow, yet short enough with respect to the integral time scale of turbulence in order to contain all the important information of the turbulent flow.

There have been numerous numerical methods proposed for the computer simulation of turbulent flow by solving the Reynolds averaged equations. Among them, the k-ω two-equation turbulent model is well suited for the low speed conditions in buoyancy cavity flows. The k-ω model is to solve the Reynolds averaged approximate Navier-Stokes equations along with two additional equations governing turbulence, which are necessary to account for the additional turbulent flow variables. The two additional turbulent flow variables are turbulent kinetic energy k, and turbulent specific dissipation rate ω.

The derived incompressible Navier-Stokes and the turbulence governing closure equations are given below:

Conservation of Mass (Continuity):

$$\partial_i U_i = 0 \qquad (1)$$

Newton's Second Law (Momentum):

$$\rho\,\partial_0(U_i) + \rho U_j \partial_j (U_i) = \partial_i P + (\mu + \mu_T)\partial_i(\partial_j U_i + \partial_i U_j) + \rho\,\beta\,g_i(T - T_o) \qquad (2)$$

First Law of Thermodynamics (Energy):

$$\rho\,(\partial_0 T + u_i \partial_i T) = \left(\frac{\mu}{Pr} + \frac{\mu_T}{\sigma_T}\right)\partial_i \partial_i T \qquad (3)$$

Turbulence Kinetic Energy:

$$\rho\partial_0 k + \rho U_j \partial_j k = \mu_T(\partial_j U_i + \partial_i U_j)\partial_j U_i - \rho k\omega + \partial_j\left[\left(\mu + \frac{\mu_T}{\sigma_k}\right)\partial_j k\right] + \beta\frac{\mu_T}{\sigma_T}\partial_i T g_i \qquad (4)$$

Specific Dissipation Rate of Turbulence Kinetic Energy:

$$\rho\partial_0 \omega + \rho U_j \partial_j \omega = \alpha'\frac{\omega}{k}\mu_T(\partial_j U_i + \partial_i U_j)\partial_j U_i - \beta'\rho\omega^2 + \partial_j\left[\left(\mu + \frac{\mu_T}{\sigma_\omega}\right)\partial_j \omega\right] + \alpha'(1-c_3)\beta\frac{\mu_T}{\sigma_T}\partial_i T g_i \frac{\omega}{k} \qquad (5)$$

Auxiliary Relations and Closure Coefficients:

$$\mu_T = \frac{c_\mu \rho k}{\omega} \quad \varepsilon = k\omega \quad u_T = k^{1/2} \quad \ell_T = \frac{k^{1/2}}{\omega} \qquad (6)$$

$$\sigma_k = 2 \quad \sigma_\omega = 2 \quad \alpha' = 5/9 \quad \beta' = 3/40 \quad c_\mu = 9/100 \quad \sigma_T = 9/10 \quad c_3 = 4/5 \qquad (7)$$

where, μ_T is the turbulent eddy viscosity, ω is the specific dissipation rate of turbulence kinetic energy, u_T is the turbulent characteristic velocity and ℓ_T is the turbulent characteristic length. The values in equations (6) and (7) are closure coefficients.

The numerical approximation techniques utilized to obtain the solutions to the governing equations described above is the Finite Element Method based on the Galerkin Weighted Residual. Nine node isoparametric quadratic elements are used throughout this study. The mesh densities used are 16x128 for aspect ratios 30 and 40, 22x160 for aspect ratios 50, 60, 70 and 80. These mesh densities were chosen, because using finer mesh only change the results of the average Nusselt number by less than 0.015%.

3. Results

Modeling of turbulent natural convection in vertical cavities of aspect ratio of 30, 40, 50, 60, 70 and 80 was completed. Table 2 shows the ranges of the Rayleigh number, which were covered in this study for each aspect ratio.

Table 2: The Range of Current Study

A	Range of *Ra* for turbulent modeling
30	17,500 to 200,000
40	15,625 to 200,000
50	15,500 to 200,000
60	15,400 to 200,000
70	15,500 to 200,000
80	15,200 to 200,000

Figure 2 to 7 show the average Nusselt number versus Rayleigh number for aspect ratio 30, 40, 50, 60, 70 and 80, respectively. The smaller figure amplifies the curves at lower laminar Rayleigh number ranges.

(1) A = 30

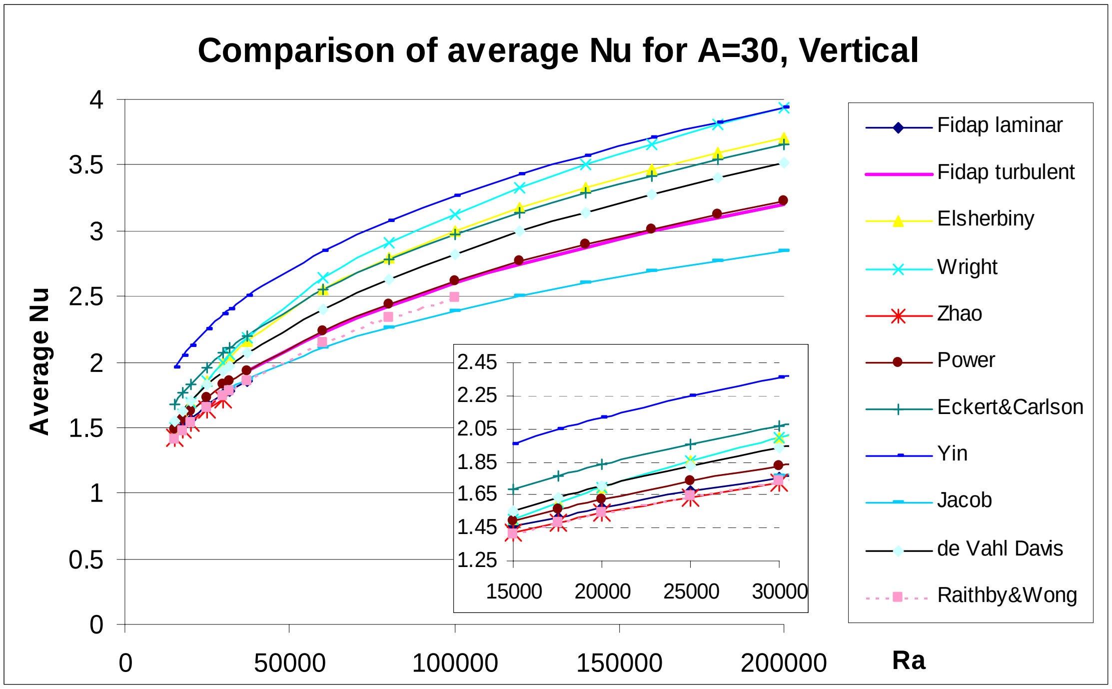

Figure 2: Average Nusselt number versus Rayleigh number for A=30

Comparing the results obtained from different researches, it can be found that all the results follow more or less the same trend. Nu increases fairly quickly with respect to Ra when Ra is small. And the slopes of the curves decrease with the increase of Ra in all the curves shown above. It is also discovered that when Ra becomes higher, for every curve, its slope tends to be a fixed value. In other words, the curves are linear at high Ra range.

Generally speaking, the Nu curve obtained in this research is lower than that from most of the researches obtained through literature. However, it is worth mentioning that current results match those of Power extremely well. It is believed that this is induced by the coincidence of modeling technique and simulation methods.

(2) A = 40

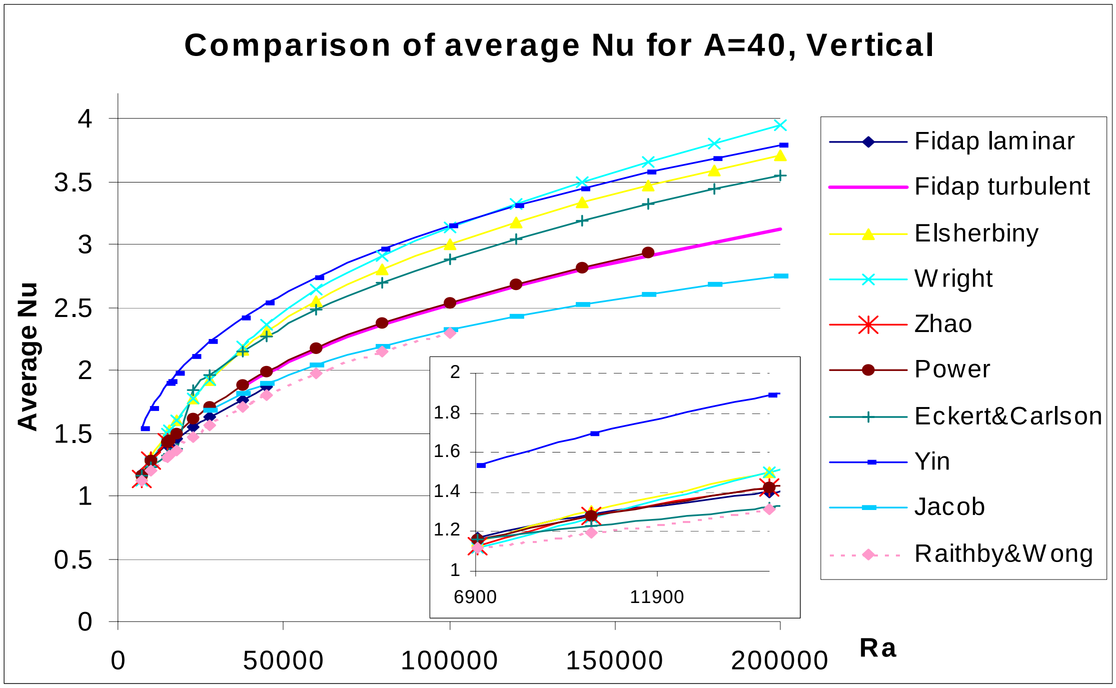

Figure 3: Average Nusselt number versus Rayleigh number for A=40

From Figure 3, it is evident that the curve follows the same trend as that of A=30. The relative position of the curve among all the curves is also very close to that of A = 30. The curves also go linear at high Ra range.

(3) A = 50

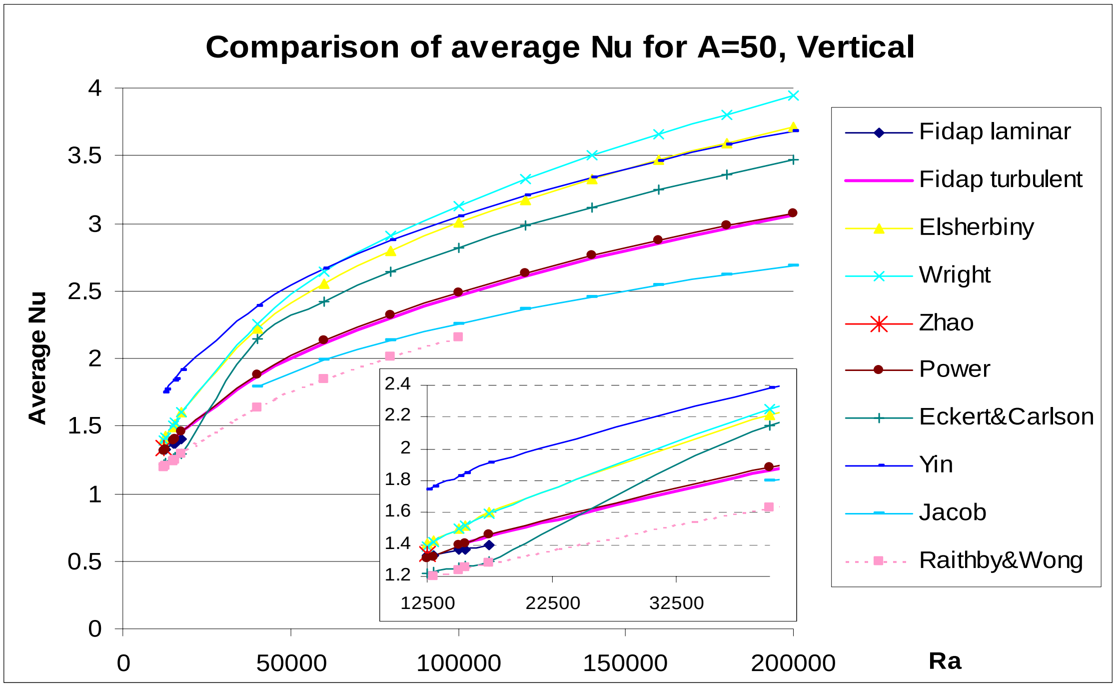

Figure 4: Average Nusselt number versus Rayleigh number for A=50

Again the basic trend of the curve of current research is very consistent with that of A=30 and A=40. The Nu keeps decreasing with the increase of aspect ratio. Also almost the same trend can be discerned for A = 60, A=70 and A=80 from Figure 5, 6 and 7 respectively.

(4) A = 60

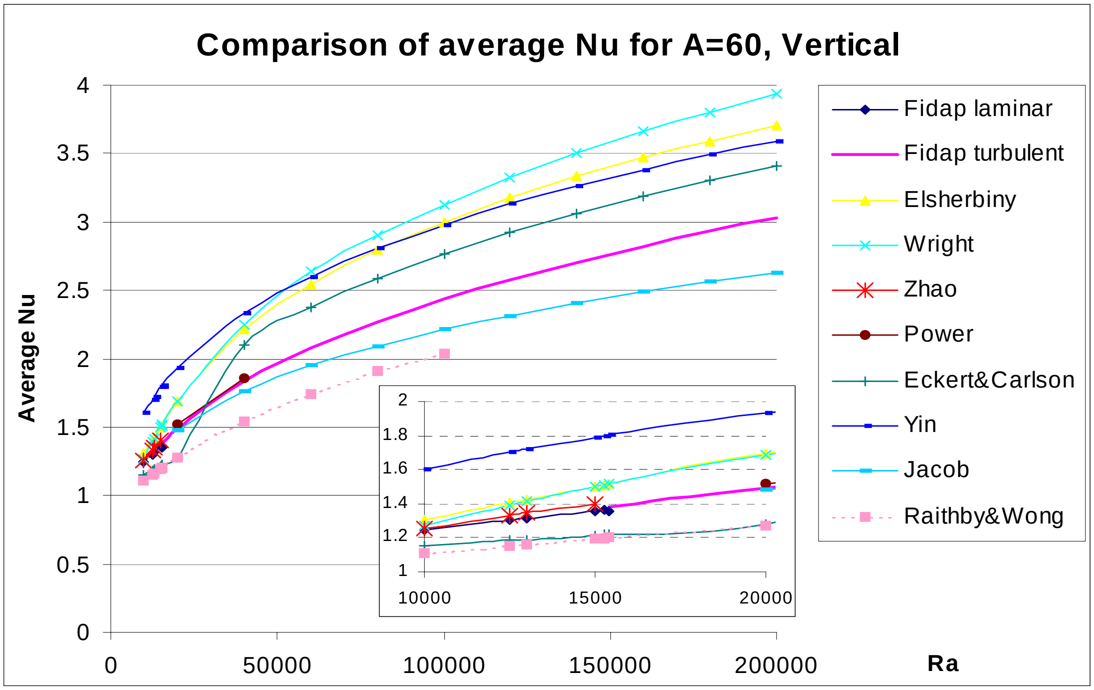

Figure 5: Average Nusselt number versus Rayleigh number for A=60

(5) **A = 70**

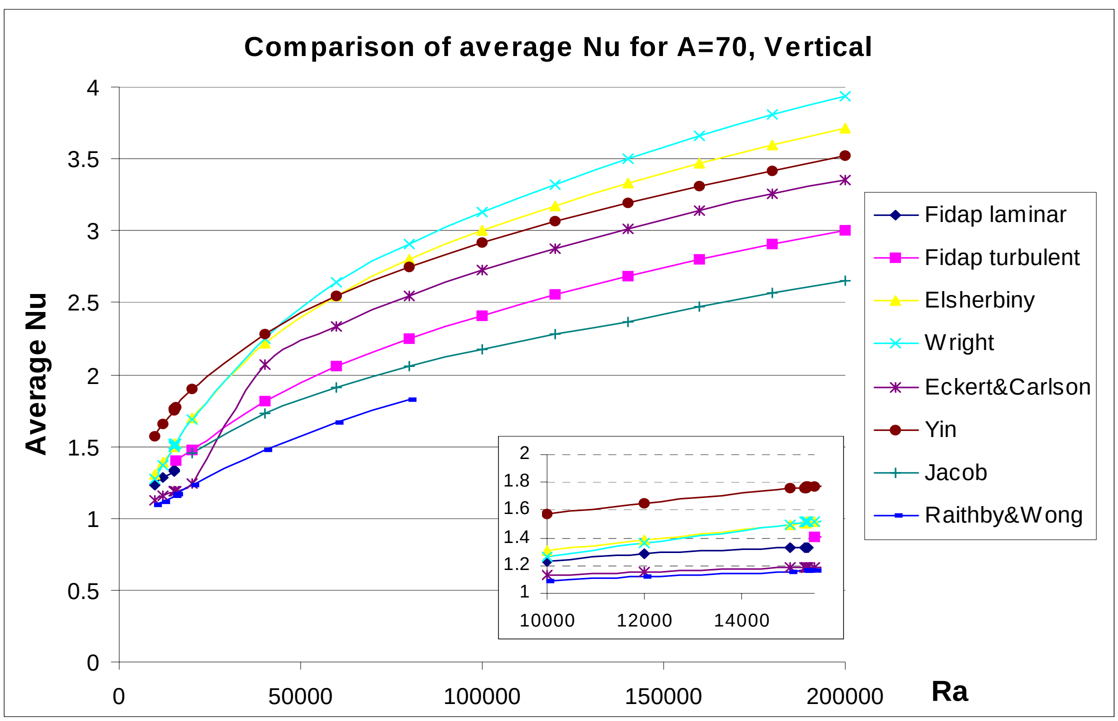

Figure 6: Average Nusselt number versus Rayleigh number for A=70

(6) A = 80

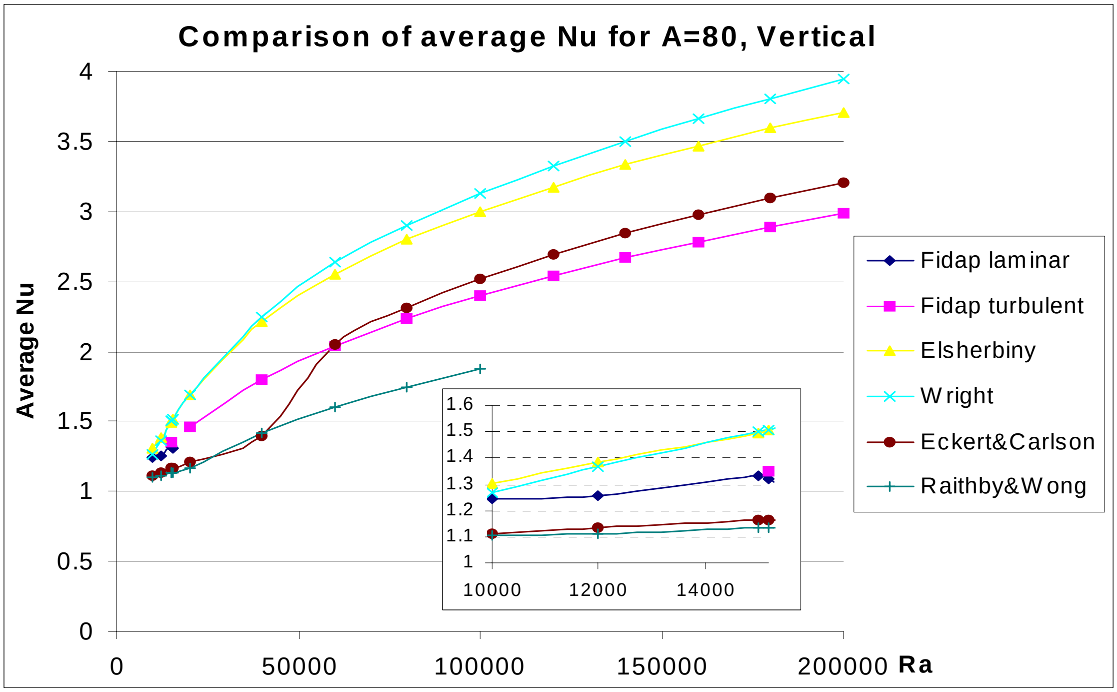

Figure 7: Average Nusselt number versus Rayleigh number for A=80

4. Results and Discussions

Due to the consistency of the data under different aspect ratios and Rayleigh numbers, some simple functions should be able to satisfy the requirement of correlation. In current research, a simple power relation was developed by the software Mathematica to correlate average Nusselt number as a function of Rayleigh number and aspect ratio. The correlation has the form:

$$Nu = 0.109082 Ra^{0.30167} A^{-0.0893116}, \quad 15200 \le \mathrm{Ra} \le 200{,}000, 30 \le \mathrm{A} \le 80, \qquad (8)$$

The curves shown in Figure 8 represent the correlation equation (8), and the differences between the calculated data from the correlation and the simulated data from Fidap are within 2% from each other. This shows that the correlation can match the data to a very precise level, which is definitely enough for usual industrial applications. So with extended design ranges in Ra and A, the two most important parameters in enclosure buoyancy flows, the correlation can still give very accurate prediction of the heat transfer inside the enclosure. This will greatly facilitate the design of fenestration systems and the major objective of the current research has been accomplished.

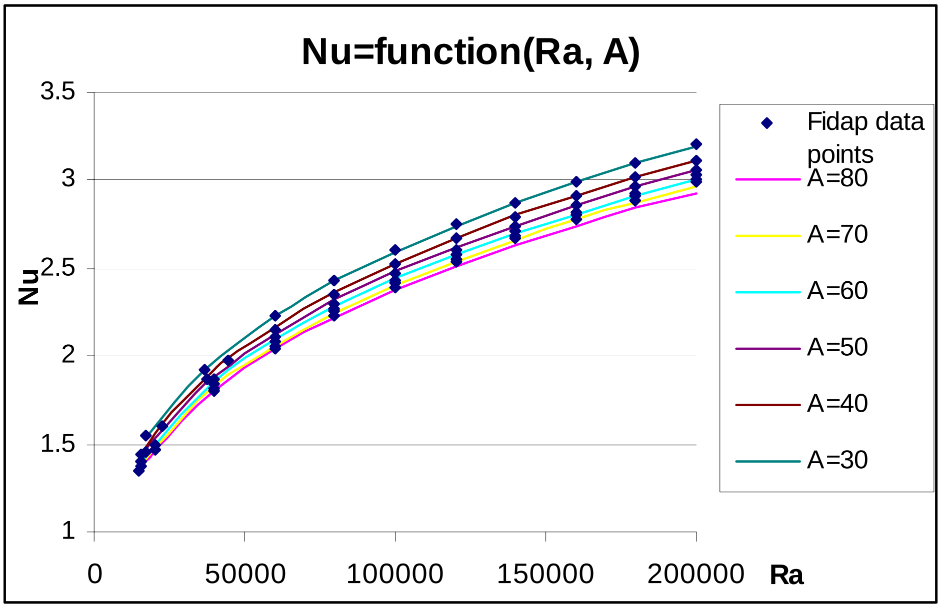

Figure 8: Nusselt number correlation compared to Fidap data

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