Turbulent Natural Convective Heat Transfer in Vertical Glazing Cavities (Draft)

Abstract—In this work, turbulent natural convective flow and heat transfer in vertical glazing cavities is studied numerically by the finite element method. A scheme that involves a statistical quantity is applied to detect the onset of turbulence in rectangular cavities with aspect ratio from 30 to 100 when flow passes the limit of transition to turbulence in multicelluar regime. From aspect ratio 20 to 30, such limit is found by solving the laminar and turbulent governing equations and comparing the difference of heat transfer results between them. Turbulent flow in enclosures with aspect ratio from 20 to 100 is then modeled for Rayleigh number up to 200,000. And a correlation of average Nusselt number is developed as a function of Rayleigh number and aspect ratio.

INTRODUCTION

Buoyancy-driven natural convection in confined cavities is receiving more and more research attention due to its wide applications, such as multi-layered walls, multi-pane windows and other air gaps in unventilated spaces. Depending on the conditions such as size and shape of the cavity and amount of heating applied, the flows in the cavity can be either laminar or turbulent. While laminar flow is relatively simple and well defined, turbulent flow represents one of most complicated phenomena in the nature. In engineering applications, turbulence often displays apparent differences from laminar flows. Comparatively speaking, turbulent flows often lead to higher transport rate of momentum, energy and mass than laminar flows. These features are widely made use of in energy systems in industry. For example, turbulence enhancers such as ribs are added to cooling systems of turbine blades and microelectronic devices to create more turbulent motions so that the overall heat transfer efficiency can be improved. However, turbulence may also be the adverse condition that people try to avoid. For instance, the enhanced heat transfer efficiency caused by turbulence in building systems usually means that more energy will be transferred between the building and its surroundings, resulting in more energy consumption of the building. For this reason, study of the characteristics of turbulent fluid flow and heat transfer is very important for more efficient utilization of building energy system. This is the motivation of this work.

In this work, the transition from laminar flow to turbulence in vertical glazing cavities was first studied. As a very complex process, there have been quite some researchers working on this topic before. Yang (1988), gave a very good summary of this process. He points out that occurrences of flow bifurcations change the structure of the flow and gradually lead to the commencement of turbulence with the change of controlling parameter such as heating. As the underlying mechanism for transition to turbulence is very complicated and the description of this process needs a deep understanding of physics and mathematics, which is not available to most of the designers in engineering field, the first purpose of this paper is to present a correlation of the limit of transition to turbulence with respect to the controlling parameters of the flow. Then, the heat transfer of turbulent flow in the cavities was studied. A simple correlation of heat transfer with respect to the controlling parameters, such as Rayleigh number and aspect ratio, will be given in the turbulent regime. These correlation equations would be easily understandable by designers and applicable as a guide in their design of building systems or other systems where flows in confined enclosures exist.

LITERATURE REVIEW

Numerous studies can be found in literature on turbulent flow. Several work provided correlations of transitional limit from laminar to turbulent regime in vertical rectangular enclosures. The earliest work can be found in Batchelor (1954). Yin et al. (1978) presented measurement data that fluctuated at sufficiently high Ra, and Wright (1990) interpreted this as an indicator of the transition to turbulence. Power (1999) used a transient method in order to observe any fluctuations of the numerical solution of average Nusselt number over time. Although these three studies all provide simple power law form of correlation for the Rayleigh number limit of transition as a function of aspect ratio, their results lack universal consistency. And the applicable aspect ratio ranges of the given correlations are not large enough and need to be extended to higher aspect ratio.

Turbulent heat transfer in vertical rectangular enclosures has also been extensively studied by both experimental and numerical work. The early experimental study of Jakob (1967) gave a correlation equation of the form

$$Nu = C(Ra)^a (A)^b \tag{1}$$

with the exponents a=1/3, and b=1/9 for turbulent flow in cavities with aspect ratio from 3.0 to 42.2. Yin et al. (1978) presented a correlation equation with a=0.269 and b=0.131 and the aspect ratio range covers up to 78.7. The experimental work of Elsherbiny et al. (1982) provided a set of correlations at aspect ratio A=5, 10, 20, 40, 80 and 110. Raithby and Wong (1981) numerically gave a single but more complicated form of correlation covering aspect ratio from 2 to 80, but with only half of the range of Rayleigh numbers in the current study, which is up to 200,000. More recently, Power (1999) numerically calculated and presented a correlation for the turbulent heat transfer in cavities with aspect ratios from 5 to 60.

Generally speaking, data on turbulent heat transfer in high aspect ratio cavities are scarce. Correlations of turbulent heat transfer presented in literature are mostly only applicable to cavities with low and medium aspect ratio. Even though there are some data available for high aspect ratio cavity flow, such as those of Elsherbiny et al. (1982), Yin et al. (1978) and Power (1999), the Rayleigh number in their studies only extended slightly beyond the limit of transition to turbulent regime for those cavities with high aspect ratios. In these studies, as aspect ratio increases, the Rayleigh number range being studied narrows.

PROBLEM DESCRIPTION

A two-dimensional rectangular cavity is shown in Fig. 1. In real application, the cavity is always three-dimensional. However, the size of the cavity in the third dimension is usually very large compared to that of the direction with applied temperature gradient. More importantly, there are seldom any effects such as temperature gradient or gravity applied in the third direction, which makes the flow in this direction very close to homogeneous. Thus, in the description of this problem, a 2-D model is usually applied to study the flow and heat transfer in the vertical cavity.

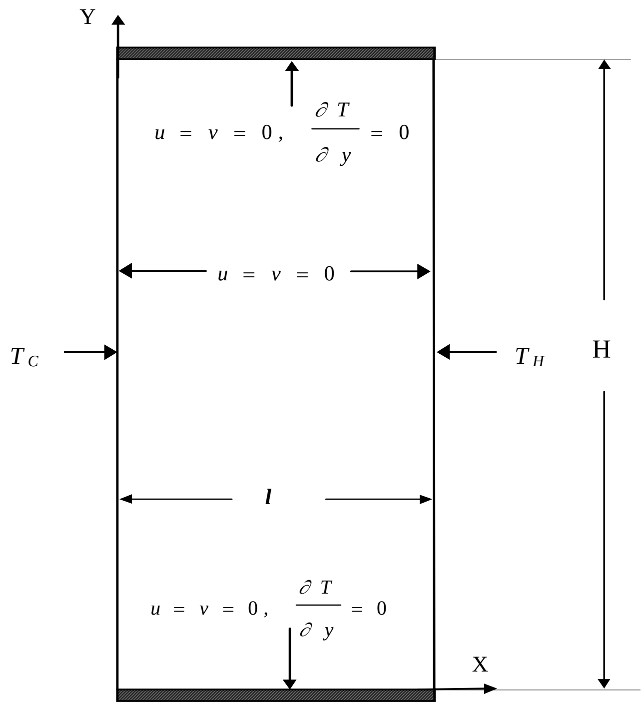


Figure 1: Schematic of a rectangular cavity.

From Fig. 1, it can be seen that the two-dimensional cavity has two horizontal sides and two vertical sides. The two horizontal sides are adiabatic and the two vertical walls have constant temperatures with $T_H > T_C$. Under this condition, the density gradient of internal fluid is normal to the gravity and natural convection starts immediately when heat is applied.

For calculation purposes, it is often convenient to non-dimensionalize the governing equations and to introduce the characteristic dimensionless numbers. The governing equations are non-dimensionalized based on the following scaling quantities:

$$u^* = \frac{u}{U}, \ v^* = \frac{v}{U}, \ U = \frac{\alpha}{L}\sqrt{Ra\Pr} = \sqrt{g\beta\Delta TL} \tag{2}$$

$$x^* = \frac{x}{L}, \ y^* = \frac{y}{L} \tag{3}$$

$$\theta^* = \frac{T - T_0}{\Delta T}, \ \Delta T = T_H - T_C \tag{4}$$

$$p^* = \frac{pL}{\mu U} \tag{5}$$

$$Ra = \frac{\rho g\beta(T_H - T_C)L^3}{\mu\alpha}, \ \Pr = \frac{\nu}{\alpha} = \frac{\mu C_p}{k} \tag{6}$$

$$t^* = \frac{tU}{L}. \quad (7)$$

The Boussinesq approximation, as well as the assumption of an incompressible fluid flow with constant properties, is applied. The governing equations in non-dimensional form are:

Continuity equation: $$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0. \quad (8)$$

Momentum equations:

x-direction:

$$\sqrt{\frac{Ra}{\Pr}}[\frac{\partial u^*}{\partial t^*} + u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*}] = -\frac{\partial p^*}{\partial x^*} + [\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}}]. \quad (9)$$

y-direction:

$$\sqrt{\frac{Ra}{\Pr}}[\frac{\partial v^*}{\partial t^*} + u^* \frac{\partial v^*}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^*}] = -\frac{\partial p^*}{\partial y^*} + [\frac{\partial^2 v^*}{\partial x^{*2}} + \frac{\partial^2 v^*}{\partial y^{*2}}] + \sqrt{\frac{Ra}{\Pr}}\theta^*. \quad (10)$$

Energy equation:

$$\sqrt{Ra\Pr}[\frac{\partial \theta^*}{\partial t^*} + u^* \frac{\partial \theta^*}{\partial x^*} + v^* \frac{\partial \theta^*}{\partial y^*}] = \frac{\partial^2 \theta^*}{\partial x^{*2}} + \frac{\partial^2 \theta^*}{\partial y^{*2}}. \quad (11)$$

The boundary conditions imposed in the dimensionless form for this problem are as follows:

Temperature boundary conditions on the side walls:

$$\theta^*(x^* = 0, y^*) = 0, \quad \theta^*(x^* = 1, y^*) = 1. \quad (12)$$

Non-slip velocity boundary conditions on all bounding surfaces are:

$$u^*(x^* = 0, y^*) = v^*(x^* = 0, y^*) = 0, \quad (13)$$

$$u^*(x^* = 1, y^*) = v^*(x^* = 1, y^*) = 0, \quad (14)$$

$$u^*(x^*, y^* = 0) = v^*(x^*, y^* = 0) = 0, \quad (15)$$

$$u^*(x^*, y^* = H^*) = v^*(x^*, y^* = H^*) = 0, \quad (16)$$

where H^* is the dimensionless height of the cavity.

Boundary conditions at the top and bottom surfaces:

ZHF(Zero Heat Flux): $$\left.\frac{\partial \theta^*}{\partial y^*}\right|_{y^*=0} = 0, \quad \left.\frac{\partial \theta^*}{\partial y^*}\right|_{y^*=H^*} = 0. \quad (17)$$

The dimensionless parameters governing the flow behavior in a vertical cavity are aspect ratio A and Rayleigh number Ra. The aspect ratio A is defined as

$$A = \frac{H}{L}, \quad (18)$$

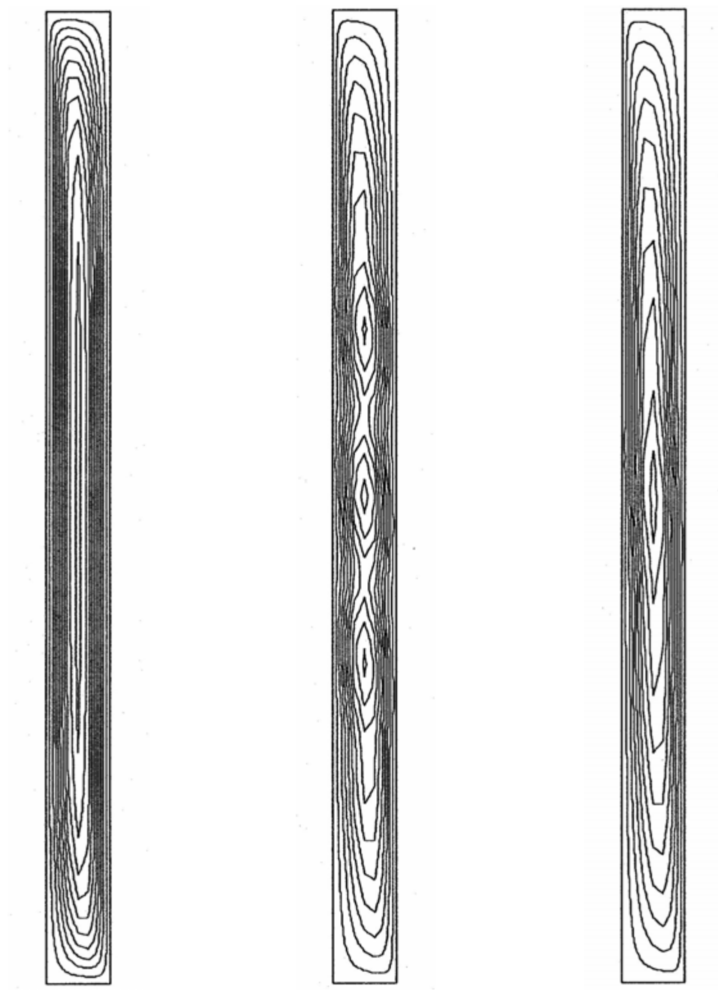
where H is the height of the cavity and L is the width of the cavity.

Prandtl number (Pr) is the property of the fluid, while Rayleigh number (Ra) incorporates the influences of fluid property, external thermal boundary condition and domain geometry into a single parameter. Based on the governing equations, it can be seen that Ra is the main determining parameter in this problem since it represents the driving force in the enclosure – buoyancy, without which there will be no turbulence in the domain. The greater the Rayleigh number is, the greater buoyancy effect will be and the flow tends to be more turbulent.

When Ra is small, which means there is only very weak buoyancy acting on the flow, it can be expected that the flow is laminar, and the convective motion of the flow creates a cell circulating inside the enclosure. The streamlines of it are presented in Fig. 2a with Ra = 5000. The aspect ratio of this cavity is 15.

As Ra increases, an important phenomenon for buoyancy driven cavity flow, multicellular condition, will occur, as shown in Fig. 2b. The Ra under this condition is 17750. There have been intensive investigations on this topic. Batchelor (1954) used linear hydrodynamic stability theory to explain the origin and characteristics of this phenomenon. Zhao (1997) applied numerical methods to predict the transitional Ra between single-cellular and multi-cellular condition with respect to aspect ratio A in the cavity described as above.

Although the flow in the cavity can become multicellular, this condition can only stay a short range of Rayleigh number for aspect ratio below 30. When the buoyancy force is further strengthened all the small cells will collapse into a single stronger cell again. This is the ending point of multicellular regime and was presented by Zhao (1997). The streamlines of this condition are shown in Fig. 2c with Ra = 28750.



(a) Ra=5000 (b) Ra=17750 (c) Ra=28750

Figure 2: Cells in a buoyancy-driven cavity with aspect ratio 15.

NUMERICAL METHODS

To solve the problem described by Equation 8 to 11 and boundary conditions 12 to 17, the finite element analysis based on the Galerkin weighted residual method is applied. Penalty function method is also used, which utilizes a fluid incompressibility constraint, leading to a reduction of the number of unknowns by eliminating the pressure from the calculations.

In calculations of transition from laminar flow to turbulence, it is a natural choice to use time-dependent scheme to solve the problem to discern the temporal fluctuations in the solutions. Another reason to apply the time-dependent method is to help the convergence of computation of the governing equations. For each enclosure with a specific aspect ratio, the principal controlling parameter, Ra, is increased incrementally to study the influence of Ra on heat transfer so that transitional point can be detected and characteristics of heat transfer can be studied. Every new calculation with different Rayleigh number and aspect ratio is initialized using the same initial condition – zero velocity and zero temperature.

A k-ω model is used when turbulence model is required. Here, k represents the turbulent kinetic energy and ω is the specific dissipation rate of turbulent kinetic energy. A detailed description of this model can be found in FIDAP (Fluent Inc., 2001).

Nine-node quadratic meshes are generated to perform the computation. The finite-element mesh employed is stretched towards the wall. The mesh density for each aspect ratio is based on the mesh study of Zhao (1997) and Power (1999).

TRANSITION FROM LAMINAR TO TURBULENT FLOW IN VERTICAL ENCLOSURES

Principles of Detecting Transitional Limits

The most important feature of turbulence is its randomness, and it is assumed that this method can be applied to find the transitional limit from laminar flow to turbulence in such a buoyancy-driven cavity. The basic underlying idea is that when the flow goes turbulent, some temporal fluctuations are created in the flow and can be detected numerically by calculating the change of overall heat transfer effect with respect to time. It is worth noting here that judging the transitional point using overall heat transfer coefficient is only applicable to flows that cross the transitional limit under multicellular condition. Due to the instability of the multicells, the overall heat transfer coefficient fluctuates temporally. As the Rayleigh Number is increased, the fluctuations of the average Nusselt number also increase. On the other hand, when the flow is single-cellular, the overall heat transfer coefficient will not change with respect to time.

For this reason, for flows entering turbulent regime under multicellular condition, after the time-dependent governing equations are solved, data are collected and overall heat transfer coefficient (average Nusselt number, Nu) with respect to time is computed and recorded to observe the fluctuation. The average Nusselt number is defined as $Nu = \frac{hL}{k} = \frac{qL}{\Delta Tk}$, which is the ratio between convection heat transfer to the conduction heat transfer, while q is the overall heat transfer rate. At lower Ra, Nu is almost a flat line with respect to time, which means there is rarely any fluctuation in the flow. However, as Ra is increased, more fluctuations in Nu can be observed corresponding to the fact that the flow tends to be more turbulent at higher Rayleigh number. An example of aspect ratio 40 is given in Fig. 3. It is easily seen that higher Rayleigh number corresponds to greater fluctuation of the average Nusselt number.

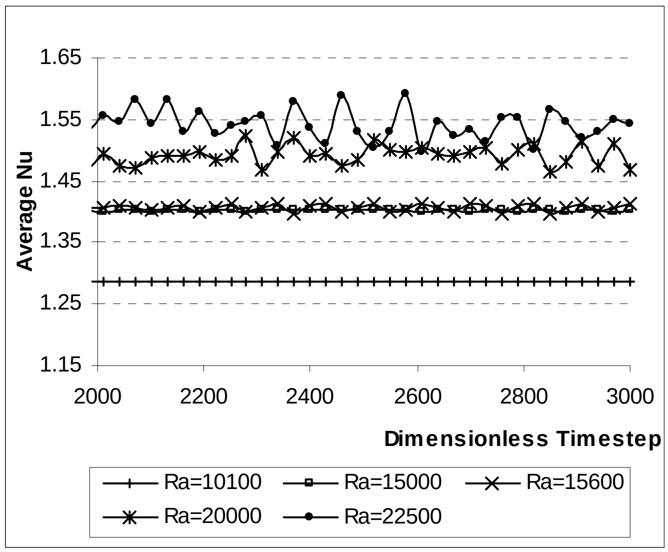


Figure 3: The average Nusselt number versus time for aspect ratio 40.

A statistical quantity, which is the ratio of the standard deviation over the mean value of average Nusselt number (Nu) over a long span of time, is used to represent the fluctuation. In Fig. 4, this quantity is plotted for each Rayleigh number of aspect ratio 40. It can be seen from Fig. 4 that there is no significant fluctuation at lower range of Rayleigh number. However, as Ra increases, a sudden increase of fluctuation level can be detected around Ra = 16,000. Further investigation shows that before the onset of the turbulence, the ratio of the standard deviation to the temporal mean value of Nu is on the order of 0.01% to 0.1%. After it reaches the transitional point, the ratio is at the level of 1%. So the flow can be judged to be turbulent if this ratio reaches 1.0%.

The method above works generally very well for cavities with a wide range of aspect ratios. However, if A is smaller than (and equal to) 30, the method is very hard to

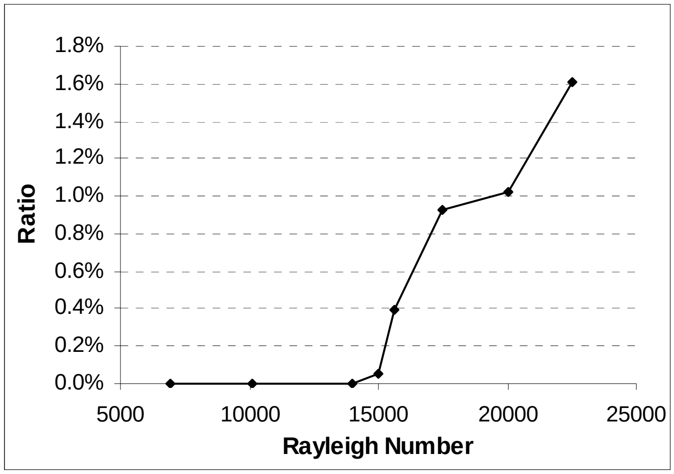


Figure 4: The ratio of standard deviation to temporal mean Nusselt number versus Rayleigh number for aspect ratio 40.

be used to detect the limit of transition. As Ra is increasing, the flow in the cavity experiences the transition from single-cell to multi-cells. However, it exits multicellular regime and returns to the single-cell condition before it reaches turbulence regime, according to Zhao (1997). This process can be observed from the variation of the standard deviation of Nusselt number in terms of Rayleigh number. An example for a cavity with A=20 is shown in Fig. 5. In other words, single cell in turbulence regime does not produce discernible fluctuations in average Nu, so that this method cannot work for cavities with A below 30.

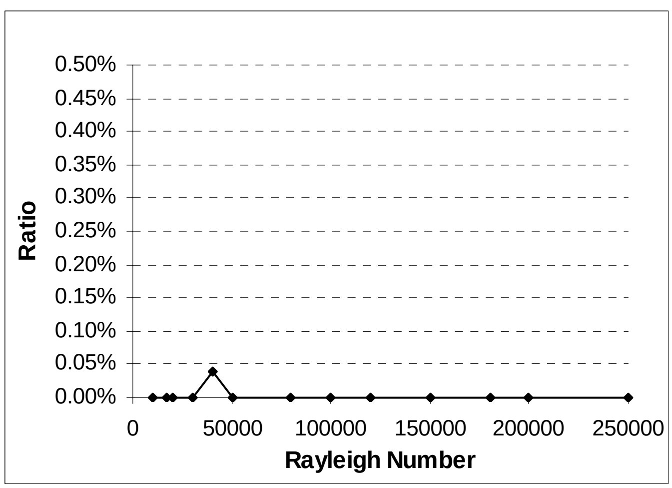


Figure 5: The ratio of standard deviation to temporal mean Nusselt number versus Rayleigh number for aspect ratio 20.

Thus, an alternative method has to be found to detect the onset of turbulence for A below 30. In this research, two sets of simulations are run to solve for Nu for each cavity with aspect ratio smaller than 30. One simulation is using laminar model. i.e. Equation 8 to 11, while the other one incorporates the $\kappa - \omega$ turbulence model mentioned above. Two results are compared and if the difference between each other is larger than 3.5%, the flow is regarded to be turbulent. The difference is defined as $(Nu_{tur} - Nu_{lam})/Nu_{tur}$.

The magnitude of the 3.5 percent difference criterion was determined by comparing Fig. 4 to Fig. 6, which shows the percentage difference between laminar and turbulent results for aspect ratio 40. From Fig. 4, the flow was judged to be turbulent when Rayleigh number is higher than 20,000, by observing the ratio of the standard deviation to temporal mean value of the average Nusselt number reaches 1.0%. Then, it is shown from the curve in Fig. 6, that Rayleigh number 20,000 corresponds to about 3.5 percent relative difference between laminar and turbulent results.

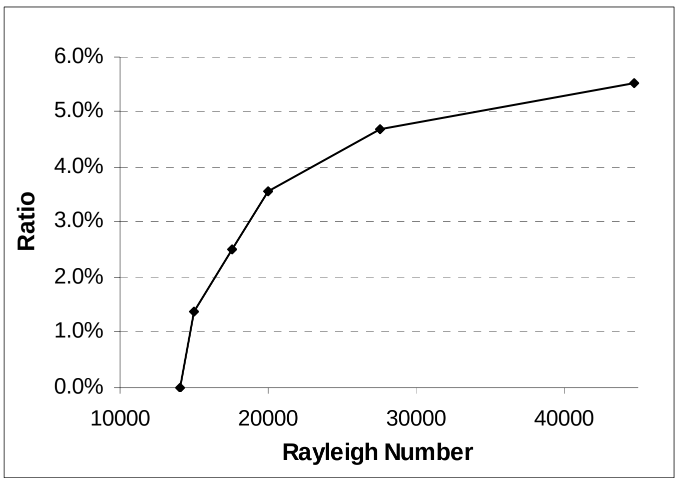


Figure 6: The relative difference between laminar and turbulent results for aspect ratio 40

Results and Analysis for Transition to Turbulence

The range of aspect ratio in current study starts from 20 to 100. When A is larger than 30, the time series of the average Nusselt number is recorded and its standard deviation is calculated. When the ratio of the standard deviation of Nu over its temporal mean value reaches 1%, the flow is then regarded to have entered the turbulent regime. An example is given in Fig. 4, which shows that for A = 40 flow turns turbulent around Ra = 20,000.

For enclosures with aspect ratio smaller than (and equal to) 30, the alternative method, which is to compare the average Nusselt number results of the laminar governing equations and the turbulent governing equations, is applied. Whenever the relative difference of the two results is over 3.5% of the turbulent value, turbulence is regarded to have occurred in the flow. Such difference for a cavity A=20 is plotted in Fig. 7, and applying the 3.5 percent criterion gives us a transitional Rayleigh number about 130,000.

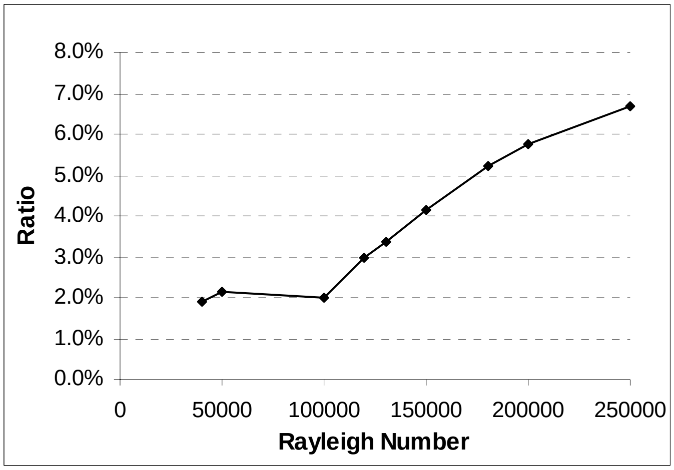


Figure 7: The relative difference between laminar and turbulent results for aspect ratio 20.

The overall results are plotted in Fig. 8, which covers aspect ratios from 20 to 100. Applying a least-squares curve fit, it is found that, for cavities with aspect ratio higher than 35.4, transitional limit remains a constant value of 21,070 approximately. Batchelor (1954) also predicted that transitional limit became independent of aspect ratio from a critical aspect ratio value of 42, which is very close to the current results, although his constant value of Rayleigh number is 13,700, around 35% lower than that of current study.

For aspect ratios less than 35.4, simulation data from current study produced a simple power law correlation with the form shown in Equation 19:

$$Ra = 3\times10^{9}\times \mathrm{A}^{-3.3285} \qquad (19)$$

This result is very close to that of Batchelor's with the same form, with coefficient 10^9 and exponent -3, as shown in Fig. 8.

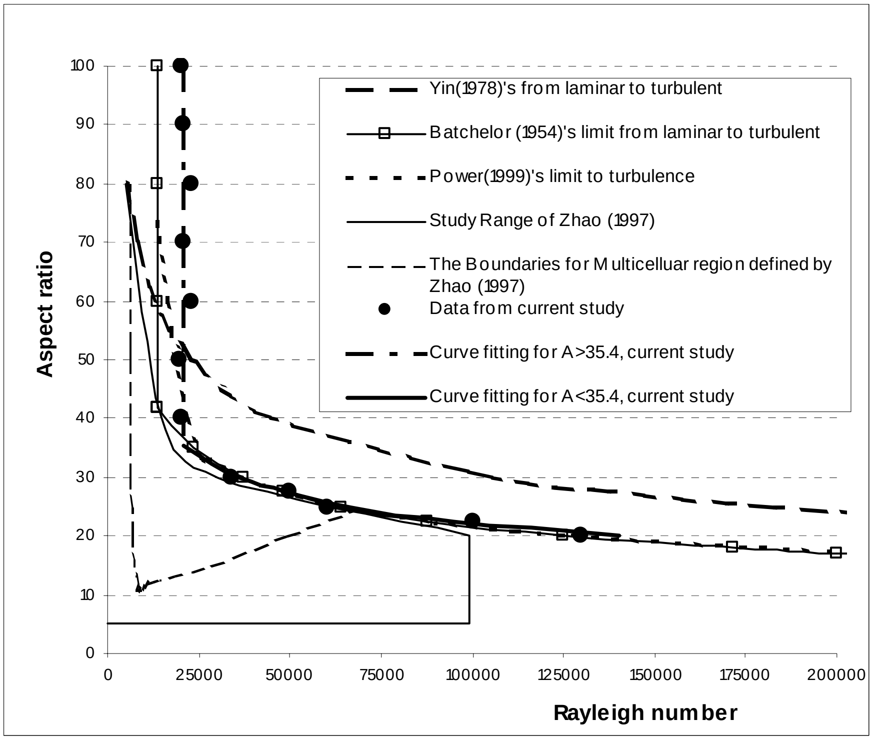


Figure 8: The limit of transition from laminar to turbulent Regime for 2-D vertical glazing cavities.

The overall correlation for the limit of transition from laminar to turbulent regime for air in the vertical glazing cavity is shown in Equation 20:

$$\begin{aligned} Ra &= 21070 && \text{for } 35.4 \leq A \leq 100 \\ Ra &= 3\times 10^{9} \times A^{-3.3285} && \text{for } 20 \leq A < 35.4 \end{aligned} \qquad (20)$$

The correlation equations given in Equation 20 agree with the individual numerically calculated results very well, with only one data point having the maximum relative difference of around 11% and rest of the data lying within the range of difference of 8.4%. And the standard deviation of the relative differences is 6.7%.

TURBULENT HEAT TRANSFER IN VERTICAL ENCLOSURES

Heat Transfer Correlation

The range of the study of turbulent heat transfer starts from the obtained results of the limit of transition from laminar to turbulent regime from current study, and ends at Rayleigh number 200,000, covering aspect ratio from 20 to 100. The heat transfer results obtained from the numerical calculation for all the aspect ratios were plotted in Fig. 9. By applying a nonlinear least-square fit, a correlation for the average Nusselt number as a function of Rayleigh number and aspect ratio has been developed, with the form of the following:

$$Nu = \frac{0.0979573 \times Ra^{0.310338}}{A^{0.0860783}} \tag{21}$$

The maximum relative difference between the simple power law form of correlation of Equation 21 and the data point obtained from simulation is 1.7%. There are 88 percent of the data points having the relative difference lower than 1%. And the standard deviation of the relative differences is 0.76%.

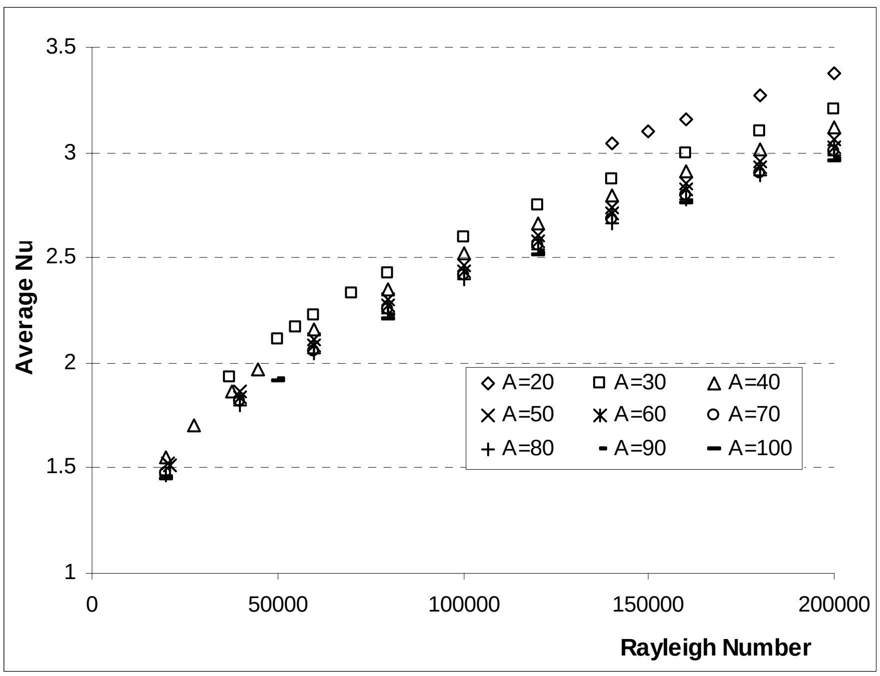


Figure 9: The average Nusselt number results for turbulent natural convection from current study.

Results and Discussions

The new correlation Equation 21 was used to compare with existing correlations derived either from experimental data or from numerical data in the literature. These existing studies are the experimental correlation from Elsherbiny et al. (1982) and Yin et al. (1978), and the numerical correlation from Raithby and Wong (1981) and Power (1999). Comparison is carried out at the aspect ratios, which are included in most of these studies, and they are A=20, 40 and 80. Figure 10 to Figure 12 show the results of the comparison.

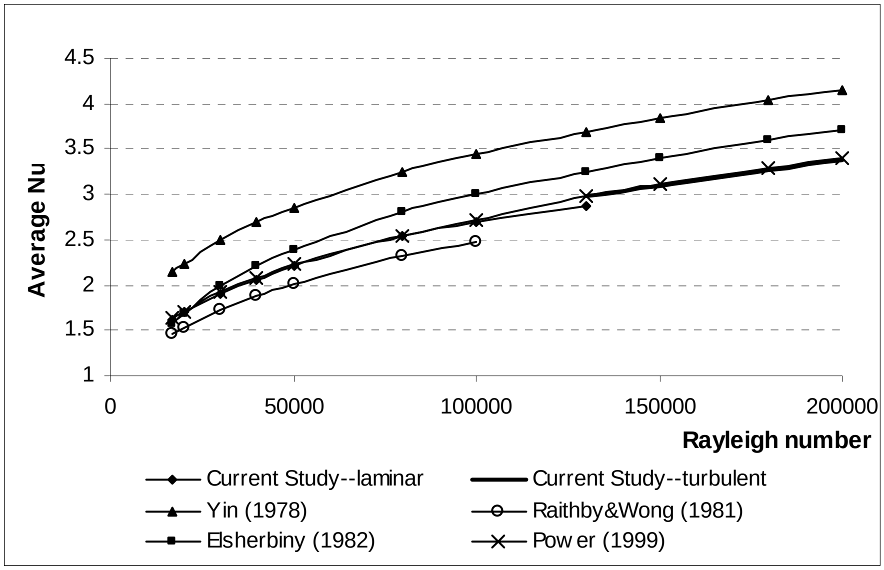


Figure 10: Comparison of the average Nusselt number for aspect ratio 20.

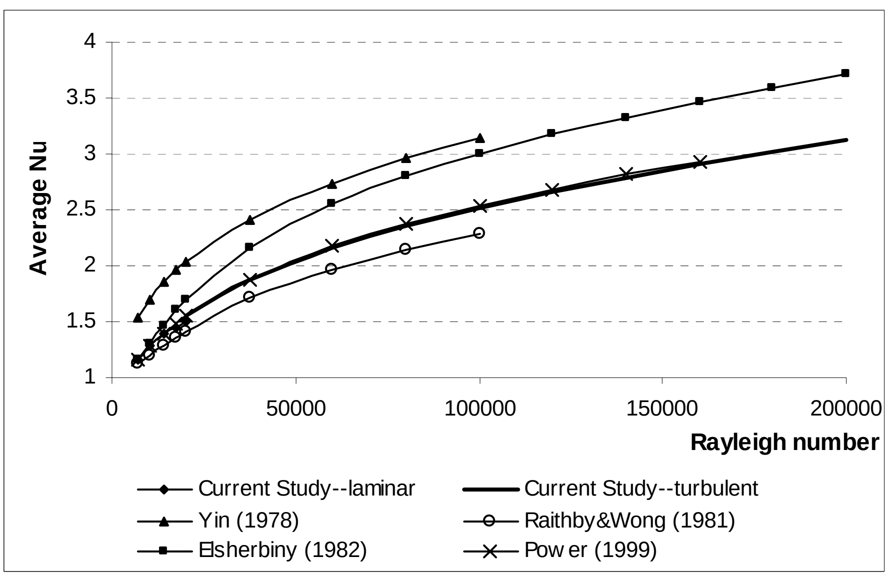


Figure 11: Comparison of the average Nusselt number for aspect ratio 40.

From Fig. 10 to Fig. 11, it can be seen that all the curves of numerical results from current study lie above the numerical results of Raithby and Wong (1981) and below the experimental results of Elsherbiny et al. (1982) and Yin et al. (1978), and agree quite well with the numerical results of Power (1999). The reason which counts for the excellent agreement between the current results and those of Power (1999) may come from applying the same numerical method the finite element method and the same software FIDAP. The major difference between current results and those of experimental studies is thought to probably result from the deviation from the ideal condition in the experimental setup.

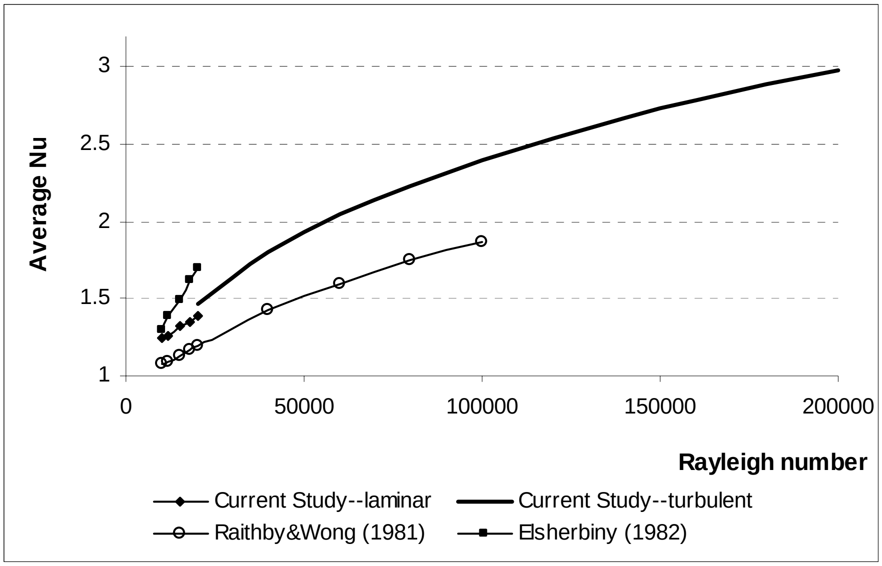


Figure 12: Comparison of the average Nusselt number for aspect ratio 80.

For example, in the experimental work of Elsherbiny et al. (1982), varying Rayleigh number was achieved by varying the pressure while keeping the dimension of the cavity constant. It is mentioned in their paper that, pressure was raised up to 0.7M Pa to obtain a Rayleigh number around 10,000 for the 6mm spacing between the hot and cold plates, which represents a cavity with aspect ratio about 100 according to the dimensions of the plates. It is roughly calculated by the author that to achieve Rayleigh number 200,000 for aspect ratio 40, the pressure was also needed to increase to around 0.7M Pa in their experiment. The high pressure would probably lead to some deviation from the ideal gas situation, and if still applying the thermo-physical properties of ideal gas, some error would be added to the measurement results. As we could see from Fig. 10 for aspect ratio 20, the difference between current study and that of Elsherbiny et al. (1982) is no larger than 8%, because pressure applied in the experiment is only a little higher than the atmospheric pressure (around 0.2M Pa for Rayleigh number 200,000). However, for aspect ratio 40 in Fig. 11, this difference increases with Rayleigh number, and reaches about 14% at Rayleigh number 200,000. Because the pressure was increased gradually to obtain an increasing Rayleigh number, and the higher the Rayleigh number, the more deflection from the ideal gas law.

There is scarcely available data for turbulent flow for aspect ratio 80 in the literature to compare with.

SUMMARY

A two-dimensional finite element model is used to study the turbulent natural convective flow and heat transfer in the glazing cavities at vertical orientation. The combination of two criteria was applied to find the limit of transition from laminar to turbulent regime for cavities with aspect ratio from 20 to 100. And the limit of transition is given as a form of a simple power law relation between Rayleigh number and aspect ratio. A correlation of the average Nusselt number as a function of Rayleigh number and aspect ratio is also developed for turbulent heat transfer in 2-D vertical glazing cavities. The range of current study covers what glazing cavities mostly encounter, with aspect ratio from 20 to 100 and with Rayleigh number from the starting of turbulence to 200,000.

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